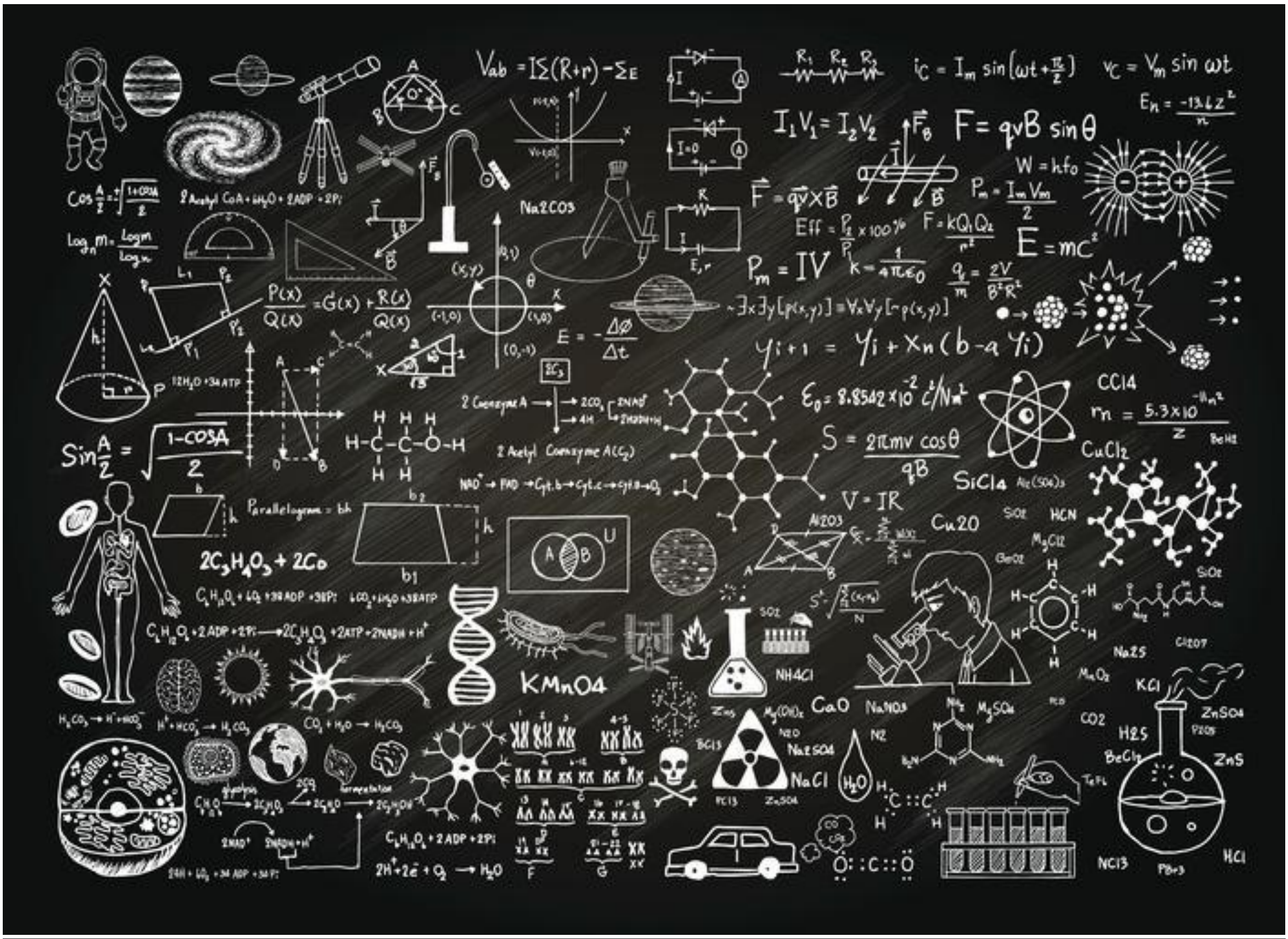
Nilasila Institute of Science and Technology, Seragarh, Balasore

APPLIED PHYSICS I

Notes For all branches of Diploma Engineering students
(Designed as per new Syllabus from session 2024-25)

Prepared by: **Miss Basumati Behera**, Lecturer in Physics,
Nilasila Institute of Science and Technology, Seragarh, Balasore
Mr. Saumyaranjan panda, Lecturer in Physics,
Nilasila Institute of Science and Technology, Seragarh, Balasore



CONTENTS

UNIT-1

PHYSICAL WORLD, UNITS AND MEASUREMENT

Physics explains the law of nature in a special way. This explanation includes a quantitative description, comparison, and measurement of certain physical quantities. To measure or compare a physical quantity we need to fix some standard unit and dimension of the quantity. In this chapter we will discuss the basic concept of Units and Dimensions and its application to various physical problems.

1.1 Physical quantities: Law of physics can be expressed through certain measurable quantities which are called as Physical quantities. Physical quantities are divided into two categories.

(1) Fundamental Quantities

(2) Derived Quantities

1.1.1 Fundamental Quantities: Fundamental quantities are those that do not depend on any other physical quantities for their measurements.

There are different systems of units which will be discussed in the coming sections. In each system of units, there are a set of defined fundamental quantities and fundamental units. Mass (M), length (L) and time(T) are some of the examples of fundamental quantities.

1.1.2 Derived Quantities: The physical quantities which are expressed in terms of other physical quantities are called as Derived Quantities.

Example- Velocity=Length/Time Acceleration=Velocity/Time=Length/$(\text{Time})^2$

Force = Mass X Acceleration = Mass X Length/$(\text{Time})^2$

1.2 Unit: Unit is a standard which is used to measure a physical quantity.

1.2.1 Fundamental Units: These units are independent and not related to each other. The units of fundamental quantities are called as Fundamental Units.

Example – The unit of length is meter. So, meter is an example of fundamental unit. Similarly, second is the fundamental unit of time and kg is the fundamental unit of mass.

1.2.2 Derived Units: The units of the physical quantities which can be expressed in terms of fundamental units are called as Derived Units. Example- Area = length X breadth = metre X metre = (metre) 2 Velocity = displacement/time=metre/second

1.2.3 Systems of Units: A complete set of units, both fundamental and derived for all physical quantities is called a system of units. The following systems of units are commonly in use.

1.2.3(A) C. G. S. System:-

This system is based on Centimetre, Gram, Second as the fundamental units of length, mass and time respectively. This system is also known as French system. In this system, unit of force is taken as dyne, unit of energy as erg, unit of power as erg/sec and unit of heat energy as calorie.

1.2.3(B) F. P. S. System:-This system is based on Foot, Pound, Second as the fundamental units of length, mass and time respectively. This system is also known as British system. In this system unit of force is taken as „poundal‟, unit of energy as „foot poundal‟, unit of power as „foot poundal/sec‟ and unit of heat energy as British thermal unit (BTU).

1.2.3(C) M. K. S. System:

This system is based on Metre, Kilogram, Second as the fundamental units of length, mass and time respectively. This system is also known as Metric system. In this system unit of force is taken as Newton, unit of energy as Joule, unit of power as Joule/sec and unit of heat energy as Joule.

1.2.3(D) SI Units: (System International d‟ Unites or International System of Units)

In C.G.S and M.K.S. system, there are three fundamental quantities e.g. mass, length, and time and accordingly three fundamental units., which are insufficient to measure some physical quantities. Therefore in 1971, the International Bureau of Weights and Measures decided a system of units which is known as International System of Units and abbreviated as SI System. It is based on the seven fundamental units and two supplementary units.

1.2.3(D)(i) Fundamental Units:

The fundamental units used to measure in SI System, are given in the following table.

Fundamental Physical Quantity	**Name of the Unit**	**Symbol of the Unit**
Mass	Kilogram	Kg
Length	Metre	m
Time	Second	s
Thermodynamic Temperature	Kelvin	K
Electric Current	Ampere	A
Luminosity	Candela	Cd
Amount of substance	Mole	mol

1.2.3(D)(ii) Supplementary units:

The supplementary units of SI System, are given in the following table.

Supplementary Physical Quantity	**Name of the Unit**	**Symbol of the Unit**
Angle	Radian	Rad
Solid angle	Ste radian	Sr

1.3.1 DIMENSIONS:

Dimensions are the power to which the fundamental units/ quantities be raised in order to represent a physical quantity.

Examples:- (1) Area = length X breadth = L X L = $[L^2] = [M^0L^2T^0]$

Here 0, 2, and 0 are the dimensions of Area with respect to mass, length and time.

(2) Velocity = Displacement/ Time = $[L]/[T] = [L^1T^{-1}] = [M^0L^1T^{-1}]$

Here 0, 1, and -1 are the dimensions of velocity with respect to mass, length and time.

1.3.2 DIMENSIONAL FORMULA:

Dimensional formula is a formula which tells us, how and which fundamental units must be used to express a physical quantity.

Dimensional formula of a derived physical quantity is the "expression showing powers to which different fundamental units are raised".

Example:-

(1) Volume (V) = length X breadth X height

$= [L] X [L] X [L]$

$= [L^3]$

$= [M^0L^3T^0]$

⇨ $V = [M^0L^3T^0]$

And 0, 3, and 0 are the dimensions of volume with respect to mass, length and time.

(2) Acceleration (a) = Velocity /Time

$= [L^1T^{-1}]/[T^1]$

$= [L^1T^{-2}]$

$= [M^0L^1T^{-2}]$

$\Rightarrow a = [M^0L^1T^{-2}]$

This is the dimensional formula of acceleration.

Here 0, 1, and -2 are the dimensions of acceleration with respect to mass, length and time.

(3) Momentum = mass X velocity $=[M^1][L^1T^{-1}] = [M^1L^1T^{-1}]$

(4) Force = mass X acceleration = $[M^1][L^1T^{-2}] = [M^1L^1T^{-2}]$

(5) Moment of a force = force X distance = $[M^1L^1T^{-2}][L^1] = [M^1L^2T^{-2}]$

(6) Work = force X distance = $[M^1L^1T^{-2}][L^1] = [M^1L^2T^{-2}]$

(7) Kinetic Energy (E_k) = ½ m v^2

$= [M^1][L^1T^{-1}]^2$ [½ has been ignored because it is dimensionless]

$= [M^1][L^2T^{-2}] = [M^1L^2T^{-2}]$

(8) Potential Energy (Ep) = m g h

$= [M^1][L^1T^{-2}][L^1]$

$= [M^1L^2T^{-2}]$

(9) Specific heat capacity (s)

$Q = ms\theta$,

where Q= heat energy

m = mass

s = specific heat capacity

θ = thermodynamic temperature.

Where s= Q/mθ

$= [M^1L^2T^{-2}]/ [M][K]$

$= [M^0L^2T^{-2}K^{-1}]$

Here 0, 2, -2 and -1 are the dimensions of specific heat capacity with respect to mass, length, time and temperature.

(10) Charge (q) = it

$= [A^1][T^1]$

$= [A^1T^1]$

Here 1 and 1 are the dimensions of charge with respect to current and time.

Sl. No	Physical Quantity	Formula	Dimensional Formula	S.I Unit
1	Area(A)	Length x Breadth	$[M^0L^2T^0]$	m^2
2	Volume(V)	Length x Breadth x Height	$[M^0L^3T^0]$	m^3
3	Density(d)	Mass/Volume	$[M^1L^{-3}T^0]$	kgm^{-3}
4	Speed(s)	Distance/Time	$[M^0L^1T^{-1}]$	ms^{-1}
5	Velocity(v)	Displacement/Time	$[M^0L^1T^{-1}]$	ms^{-1}
6	Acceleration(a)	Change in velocity/Time	$[M^0L^1T^{-2}]$	ms^{-2}
7	Acceleration due to gravity(g)	Change in velocity/Time	$[M^0L^1T^{-2}]$	ms^{-2}
8	Linear momentum(p)	Mass x Velocity	$[M^1L^1T^{-1}]$	$kgms^{-1}$
9	Force(F)	Mass x Acceleration	$[M^1L^1T^{-2}]$	N(Newton) ($kgms^{-2}$)
10	Work(W)	Force. Displacement	$[M^1L^2T^{-2}]$	J (Joule) (kgm^2s^{-2})
11	Energy(E)	Work	$[M^1L^2T^{-2}]$	J
12	Impulse(I)	Force x Time	$[M^1L^1T^{-1}]$	Ns
13	Pressure(P)	Force/Area	$[M^1L^{-1}T^{-2}]$	Nm^{-2}
14	Power(P)	Work/Time	$[M^1L^2T^{-3}]$	W(Watt)

15	Universal constant of gravitation (G)	Force x(Distance) ²(Mass)²	$[M^{-1}L^{3}T^{-2}]$	Nm^2kg^{-2}
16	Thrust(F)	Force	$[M^1L^1T^{-2}]$	N
17	Tension(T)	Force	$[M^1L^1T^{-2}]$	N
18	Stress	Force/Area	$[M^1L^{-1}T^{-2}]$	Nm^{-2}
19	Strain	Change in dimension/Original dimension	No dimensions $[M^0L^0T^{-0}]$	No unit
20	Angle (θ) Angular displacement	Arc length/Radius	No dimensions $[M^0L^0T^{-0}]$	Rad
21	Angular velocity(ω)	Angle/Time	$[M^0L^0T^{-1}]$	$rads^{-1}$
22	Angular acceleration(α)	Angular velocity/Time	$[M^0L^0T^{-2}]$	$rads^{-2}$
23	Wavelength(λ)	Length of a wavelet	$[M^0L^1T^0]$	M
24	Frequency(f)	Number of vibrations/secondor1/time period	$[M^0L^0T^{-1}]$	Hzors^{-1}
25	Angular momentum(J)	Moment of inertia x Angular velocity	$[M^1L^2T^{-1}]$	kgm^2s^{-1}
26	Temperature	Temperature	$[M^0L^0T^0K^1]$	K
27	Heat(Q)	Energy	$[M^1L^2T^{-2}]$	J
28	Latent heat(L)	Heat/Mass	$[M^0L^2T^{-2}]$	Jkg^{-1}
29	Specific heat(S)	$\frac{\text{Heat}}{\text{Mass} \times \text{temperature}}$	$[M^0L^2T^{-2}K^{-1}]$	$J\ kg^{-1}K^{-1}$
30	Thermal expansion Coefficient or thermal expansivity	$\frac{Changeindiemnsion}{originaldimension \times temperature}$	$[M^0L^0T^0K^{-1}]$	K^{-1}
31	Thermal conductivity	$\frac{HeatEnergy \times thickness}{Area \times temperature}$	$[M^1L^1T^{-3}K^{-1}]$	$Wm^{-1}K^{-1}$
32	Charge (q)	Current x time	$[M^0L^0T^1A^1]$	C (Coulomb)
33	Electric potential(V), voltage, electromotive force	Work/Charge	$[M^1L^2T^{-3}A^{-1}]$	V(Volt)
34	Resistance(R)	Potential difference/Current	$[M^1L^2T^{-3}A^{-2}]$	Ω ohm
35	Capacitance	Charge/potential difference	$[M^{-1}L^{-2}T^4A^2]$	F (Farad)
36	Electrical resistivity or (electrical conductivity)$^{-1}$	$\frac{Resistance \times Area}{length}$	$[M^1L^3T^{-3}A^{-2}]$	Ω m
37	Electric field(E)	Force/Charge	$[M^1L^1T^{-3}A^{-1}]$	NC^{-1}
38	Electric flux	Electric field X area	$[M^1L^3T^{-3}A^{-1}]$	Nm^2C^{-1}
39	Electric field strength or Electric intensity	Potential difference/distance	$[M^1L^1T^{-3}A^{-1}]$	Volt/meter orNC^{-1}
40	Magnetic field (B), magnetic flux density, Magnetic induction	$\frac{Force}{current \times length}$	$[M^1L^0T^{-2}A^{-1}]$	T (Tesla)
41	Magnetic flux(Φ)	Magnetic field X area	$[M^1L^2T^{-2}A^{-1}]$	W b (Weber)
42	Kinetic energy	½XMassx(Velocity)²	$[M^1L^2T^{-2}]$	J
43	Potential energy	Mass X acceleration due to gravity X height	$[M^1L^2T^{-2}]$	J

44	Efficiency	Output work or energy / Input work or energy	No dimensions $[M^0L^0T^0]$	N o No unit
45	Permittivity of free space	Charge x Charge / Electric force x Distance x Distance	$[M^{-1}L^{-3}T^4A^2]$	Fm^{-1}
46	Permeability of free space	Force x Distance / current x current x length	$[M^1L^1T^{-2}A^{-2}]$	NA^{-2}
47	Refractive index	Speed of light vacuum / Speed of light in medium	No dimensions $[M^0L^0T^0]$	No unit

1.4.1 Dimensional Equation:

When the dimensional formula of a physical quantity is expressed in the form of an equation by writing the physical quantity on the left hand side and the dimensional formula on the right hand side, then the resultant equation is called Dimensional equation.

Example:- Work(W) = Force X displacement

$= [M^1L^1T^{-2}][L^1]$

$= [M^1L^2T^{-2}]$

$=> W = [M^1L^2T^{-2}]$ (i)

This equation is known as dimensional equation

1.4.2 Use of dimensional analysis:

Dimensional analysis has following three uses.

(i) To convert the value of a physical quantity from one system to another.

(ii) To derive a relation between various physical quantities.

(iii) To check the correctness of a given relation.

1.4.3 Principle of Homogeneity:

It states that the dimensional formula of every term on both sides of a correct relation must be same.

OR,

The dimensions of each of the terms of a dimensional equation on both sides should be the same

OR,

The dimensions of each of the terms of a dimensional equation on both sides should be the same.

1.4.4 To convert the value of a physical quantity from one system to another:

Using dimensional analysis, we can find the numerical value of a physical quantity in any system if the numerical value is known in another system.

Example:- convert a work of 5 Joule in to erg.

Solution:-

M.K.S. System-I	Physical quantities	C G S System-II
M_1=1 kg	$W=[M^1L^2T^{-2}]$	M_2=1 g
L_1=1 m	a=1	L_2=1 cm
T_1=1 s	b=2	T_2=1 s
n_1= 5J	c=-2	n_2= ?

Physical Quantities of system 1= $n_1[M_1^aL_1^bT_1^c]$ ---------------------------(1)

Physical Quantities of system 2= $n_2[M_2^aL_2^bT_2^c]$ ----------------------------(2)

Dividing equation (1) by equation (2) we get

$n_2=n_1[M_1/M_2]^a[L_1/L_2]^b[T_1/T_2]^c$

$=5 \times [1kg/1g]m/1cm]^2[1s/1s]^{-2}$

$=5 \times [1000/1][100/1]^2=5 \times 10^7$erg

Hence, work done of 5 joule $=5 \times 10^7$ erg

1.4.5 To derive a relation between various physical quantities by the method of dimensional analysis:

Relation of one physical quantity with others can be derived when the factors on which this quantity depends are known to us.

Example:- Obtain an expression for centripetal force required to move a body of mass 'm', with velocity 'v' in a circle of radius 'r'

Solution:- Let the centripetal force depend upon m, v, r as follows:

Mathematically, it can be written as

$$F \propto m^a$$
$$\propto v^b$$
$$\propto r^c$$

Here a, b, c are dimensionless constants.

$$F \propto m^a v^b r^c$$
$$=> F = k\, m^a v^b r^c \text{ ------------(i)}$$

Here k is a dimensionless constant.

Writing the dimensional formulae of physical quantities in above equation, we get

$$[M^1L^1T^{-2}]=[M^1][L^1T^{-1}]^b[L^1]^c$$

$$=>[M^1L^1T^{-2}]=[M^aL^{b+c}T^{-b}]$$

Now using principle of homogeneity, we get

$a = 1$ ....(ii)

$b + c = 1$ ....(iii)

$-b = -2$ ....(iv)

From equation (iv), b=2,

Now substituting the value of b in equation (iii),we get c=-1.

Putting the values of a=1, b=2, c=-1 in equation (i),

We get

$$\mathbf{F = k\,mv^2/r}$$

This is the required expression.

1.5 Checking the dimensional correctness of Physical relations:

To check the correctness of a relation, we find the dimensional formula of every term on both sides of the relation. If the dimensions are same then the relation is said to be dimensionally correct.

Example (1):- To check the correctness of given relation.

$$S = ut + \frac{1}{2}at^2$$

Solution:- Given relation is $S = ut + \frac{1}{2}at^2$

Dimensional formula of s=Displacement=$[L^1]$=$[M^0L^1T^0]$ (i)

Dimensional formula of u t=$[M^0L^1T^{-1}][T^1]=[M^0L^1T^0]$ ------------------------------------(ii)

Dimensional formula of $1/2\ at^2=[M^0L^1T^{-2}][T^2]=[M^0L^1T^0]$ -------------------- (iii)

From the above equations we get dimensional formula of every term are same.
Therefore, according to Principle of Homogeneity the given relation is dimensionally correct.

Example(2):-To check the correctness of given relation.

$$v=u+at$$

Solution:-Given relation is $v=u+at$

Dimensional formula of v= final velocity=$[M^0L^1T^{-1}]$ --------------------------------(i)

Dimensional formula of u=initial velocity=$[M^0L^1T^{-1}]$ --------------------------------------- (ii)

Dimensional formula of at=$[M^0L^1T^{-2}][T^1]=[M^0L^1T^{-1}]$----------------------------------(iii)

From the above equations we get dimensional formula of every term are same.
Therefore, according to Principle of Homogeneity the given relation is dimensionally correct.

Example(3):-To check the correctness of given relation.

$$v^2 - u^2 = 4as$$

Solution:-Given relation is $v^2 - u^2 = 4as$

Dimensional formula of $v^2 = [M^0L^1T^{-1}]^2 = [M^0L^2T^{-2}]$-------------------------------(i)

Dimensional formula of $u^2 = [M^0L^1T^{-1}]^2 = [M^0L^2T^{-2}]$-- (ii)

Dimensional formula of 2as$= [M^0L^1T^{-2}][L^1] = [M^0L^2T^{-2}]$ ------------------------------- (iii)

From the above equations we get dimensional formula of every term are same.
Therefore, according to Principle of Homogeneity the given relation is dimensionally correct.

1.6 Measurement:

The combination between the numerical value and name of unit is known as measurement.

Example: 1kg, 2l, 3 m etc.

1.6.1 Measuring Instrument:

It is an instrument that shows the degree or the extend or the quantity of something that we normally observe around us. These are numerous measuring instrument as vernier calipers, micrometer, screw gauge, multi meter and electronics sensor etc.

Least Count:

Least measurement of an instrument is known as least count.

Least Count (L.C.) = 1 M. S. D.- 1 V. S. D.

where M.S.D.= Main scale Division

V. S. D. = Vernier Scale Division

1.6.2 Errors in Measurement:

(i)Systematic Error: It is unidirectional either positive and negative. There are three types of systematic errors. (i) Instrumental errors (ii) Imperfection errors (iii) Personal errors

Systematic errors can be better understood if we divide them into subgroups.

They are:

- Environmental Errors
- Observational Errors
- Instrumental Errors

Environmental Errors: This type of error arises in the measurement due to the effect of the external conditions on the measurement. The external condition includes temperature, pressure, and humidity and can also include an external magnetic field. I

f you measure your temperature under the armpits and during the measurement, if the electricity goes out and the room gets hot, it will affect your body temperature, affecting the reading.

Observational Errors: These are the errors that arise due to an individual's bias, lack of proper setting of the apparatus, or an individual's carelessness in taking observations. The measurement errors also include wrong readings due to Parallax errors.

Instrumental Errors: These errors arise due to faulty construction and calibration of the measuring instruments. Such errors arise due to the hysteresis of the equipment or due to friction. Lots of the time, the equipment being used is faulty due to misuse or neglect, which changes the reading of the equipment.

The zero error is a very common type of error. This error is common in devices like Vernier callipers and screw gauges. The zero error can be either positive or negative. Sometimes the scale readings are worn off, which can also lead to a bad reading.

Instrumental error takes place due to :

- An inherent constraint of devices
- Misuse of Apparatus
- Effect of Loading

(ii)Random Errors: It is those errors which occur at irregular periods and therefore are random for sign and size these errors can arise due to an random and predictable fluctuation in experimental conditions, personal errors by the observe in taking readings etc.

Those errors, which occur irregularly and hence are random. These can arise due to random and unpredictable fluctuations in experimental conditions (Example: unpredictable fluctuations in temperature, voltage supply, mechanical vibrations of experimental set-ups, etc, errors by the observer taking readings, etc. For example, when the same person repeats the same observation, he may likely get different readings every time.

(iii) Gross Errors : This category basically takes into account human oversight and other mistakes while reading, recording, and readings. The most common human error in measurement falls under this category of measurement errors.

For example, the person taking the reading from the meter of the instrument may read 23 as 28.

Gross errors can be avoided by using two suitable measures, and they are written below:

- Proper care should be taken in reading, recording the data. Also, the calculation of error should be done accurately.
- By increasing the number of experimenters, we can reduce the gross errors. If each experimenter takes different readings at different points, then by taking the average of more readings, we can reduce the gross errors.

1.6.3 Error in calculation:

(i) Absolute error: It is defined as the absolute value of the difference between the measured value and the true value of a measurement and is usually given as the maximum possible error given a measuring tool's degree of accuracy and has the same units as the measurement.

The difference between the measured value of a quantity and its actual value gives the absolute error. It is the variation between the actual values and measured values.It is given by

Absolute error = |VA-VE|

Example: the MAE of 5 kg indicates that, on average, the measurements deviate from the true weight by 5 kg.

(ii) Percentage error: It is another way of expressing the error in measurement. This calculation allows us to gauge how accurate a measured value is with respect to the true value. Per cent error is given by the formula

Percentage error (%) = (VA-VE) / VE) x 100

(iii)Relative error: It refers to a way of measuring the difference between an estimated or approximated value and the actual value, expressed as a ratio of the absolute difference to the actual value. The ratio of the absolute error to the accepted measurement gives the relative error. The relative error is given by the formula:

Relative Error = Absolute error / Actual value

(iv)Propagation of error: It refers to the methods used to determine how the uncertainty in a calculated result is related to the uncertainties in the individual measurements.

Example: A physical quantity *x is* given by *x*

$$x = \frac{a^2 b^2}{c\sqrt{d}}.$$

If the percentage errors of measurement in a, b, c and d are 4%, 2%, 3% and 1% respectively then calculate the percentage error in the calculation of *x.*

Answer:

Given $x = \frac{a^2 b^3}{c\sqrt{d}}$

The percentage error in *x* is given by

$$\frac{\Delta x}{x} \times 100 = 2\frac{\Delta a}{a} \times 100 \;+3\frac{\Delta b}{b} \times 100$$

$$+\frac{\Delta c}{c} \times 100 + \frac{1}{2}\frac{\Delta d}{d} \times 100$$

$$= (2 \times 4\%) + (3 \times 2\%) + (1 \times 3\%) + (½ \times 1\%)$$

$$= 8\% + 6\% + 3\% + 0.5\%$$

1.6.4: Instrumental error takes place due to :

(i)An inherent constraint of devices

(ii)Misuse of Apparatus

(iii)Effect of Loading

1.6.5: How To Reduce Errors In Measurement :

Keeping an eye on the procedure and following the below listed points can help to reduce the error.

- Make sure the formulas used for measurement are correct.
- Cross check the measured value of a quantity for improved accuracy.
- Use the instrument that has the highest precision.
- It is suggested to pilot test measuring instruments for better accuracy.
- Use multiple measures for the same construct.
- Note the measurements under controlled conditions.

1.6.6 Error estimation:

To estimate the error in a measurement, we need to know the expected or standard value and compare how far our measured values deviate from the expected value. The absolute error, relative error, and percentage error are different ways to estimate the errors in our measurements.

The error can be estimated as an absolute error, a percentage error, or a relative error. The absolute error measures the total difference between the value you expect from a measurement (X_0) and the obtained value (X_{ref}), equal to the absolute value difference of both

$$\mathbf{Abs = | X_0 - X_{ref} |.}$$

Calculate Error Rate

1. Formula: Error Rate (%) = (Number of Errors / Total Picks or Orders) × 100.
2. Example: If there are 20 errors out of 2,000 picks, the calculation would be:

1.6.6 Error significant figures: Error analysis and significant figures are the two terms you might encounter when you read about measurement uncertainty. Error analysis considers the sources of error, and significant figures give a measure of how much error is to be considered in certain calculations.

Whenever you make a measurement**,** the number of meaningful digits that you write down implies the error in the measurement.

For example if you say that the length of an object is 0.428 m, you imply an uncertainty of about 0.001 m.

The number of significant figures gives a rough approximation of the fractional error in the measurement. The unit size of the least significant digit is an indication of the absolute error.

In the first example above, the unit size of the least significant digit is 0.01; this corresponds to the reported absolute error. The reported fractional error, then, is 0.01/1.70 = 0.6%

When the multipliers have different numbers of significant figures, the smallest is used. Put another way, the fractional error in the product will be of the same order of magnitude as the largest of the fractional errors in the numbers you started with.

Thus,0.3526 x 1.2 = 0.42 (not 0.42312). This same method should be used for division.

Addition is different. Consider the example: 0.2056 + 14.25 + 576.1 = 593.1. An answer of 593.1276 is not appropriate because the last three digits (.0276) add nothing to the accuracy of the results, since one of the numbers being added (576.1) is accurate only to tenths. Here the absolute error in the sum will be of the same order of magnitude as the largest of the absolute errors in the original numbers. Subtraction works similarly.

!.6.7 Significant Digits :

The objective of significant figures for numbers is to take the meaning which contributes to its measurement resolution:

- The number 14.3 is included with three significant digits. All the time significant digits are known as the non-zero digits.
- 6.14134 possess 6 significant digits. Here, all the numbers offer useful information. Also, 59 have two significant digits, and 78.3 have three significant digits.
- 1000 has only one significant digit as the one is remarkable; we don't recognize anything certainly about the units, tens, and hundreds of places, but the place holders are the zeroes in that number.
- It is also the same with the number having a decimal given as 0.00028, which contains 2 significant digits i.e., only the 2 and 8 tell us something. The total availability of zeros is only the placeholders, and help to aid the information about approximate size.
- Two thousand five (2005) has 4 significant digits i.e. the two and five are significant and we need to sum the zeroes as they're between 2 the significant digits

1.6.8 Rules for counting significant figures in a number:

- The leftmost non-zero digit is the first significant figure.
- If there is no decimal point, the rightmost non-zero digit is the last significant figure.
- If there is a decimal point, the rightmost digit is significant, zero or not.
- Any digits between significant figures are also significant.

1.6.9 Rules for Significant Figures:

The number of significant figures can be determined in a number by using these 3 rules:

- It the zeros are between two significant digits will also significant
- In the decimal, a last zero or behind zeros share is significant only
- Non zero digits are always significant

1.Rules for Addition and Subtraction Use are as Follows:

- Add or subtract in the standard way In the decimal, portion calculates the number of significant figures only of each number in the problem
- To the right of the decimal, the final answer may have no extra significant figures than the MINIMUM number of significant figures in any number in the problem.

2.Rules for Multiplication and Division are as Follows:

- The MINIMUM amount of significant figures in any number of the problem governs the number of significant figures in the answer.
- It implies that you have to distinguish significant figures to use this rule.

UNIT-II

FORCE AND MOTION

2.1. SCALAR AND VECTOR QUANTITIES:

2.1.1. Scalar quantity:

A scalar quantity is defined as the physical quantity with only magnitude and no direction. Such physical quantities can be described just by their numerical value without directions. The addition of these physical quantities follows the simple rules of algebra, and here, only their magnitudes are added.

Examples of Scalar Quantities: Mass, Speed, Distance, Time, , Volume, Density, Temperature

2.1.2. Vector quantity:

A vector quantity is defined as the physical quantity that has both directions as well as magnitude.

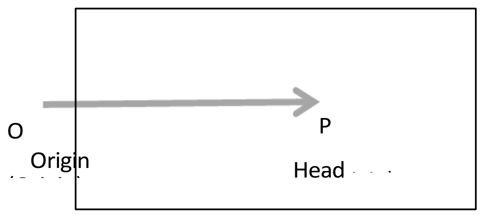
Examples of vector quantity include: Linear momentum, Acceleration, Displacement, Momentum, Angular velocity, Force, Electric field, Polarization

2.1.3 Represent a vector:

A vector "A" (Fig. 2.1) can be represented by an arrow 'OP' of finite length directed from initial point O to the terminal point P. The length of arrow represents the magnitude of vector and the arrow head denotes the direction of the vector.

A vector is written with an arrow head over its symbol like $\vec{A}$.

The magnitude of given vector is represented by modulus of vector ($|\vec{A}|$) or simply 'A'.



.

[Fig 2.1: Vector representation]

2.1.3 Types of vectors:

There are different types of vectors.

2.1.3. 1.Null vector:

It is a vector having zero magnitude and an arbitrary direction. It is represented by a point or dot (•). When a null vector is added or subtracted from a given vector, the resultant vector is same as the given vector. Dot product of a null vector with any other vector is always zero. Cross product of a null vector with any other vector is also a null vector.

2.1.3.2. Unit Vector:-

Any vector whose magnitude is one unit is called as a unit vector.

A unit vector only specifies the direction of given vector.

A unit vector in the direction of vector $\vec{A}$ is written as Â and is read as 'A cap'.

So $\vec{A}$=ÂA,

Â= $\vec{A}$/A

In three dimensional co-ordinate system , unit vectors along positive X, Y and Z-axes are usually represented by $\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$ respectively.

These unit vectors are mutually perpendicular to each other (Fig.2.2).

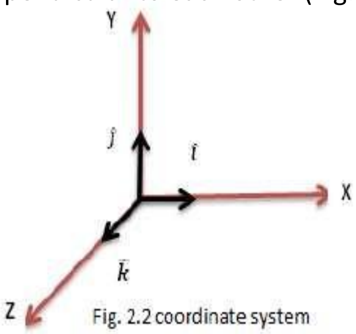


Fig. 2.2 coordinate system

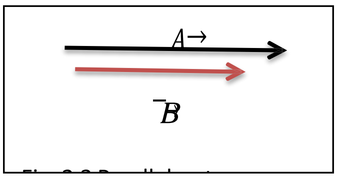
2.1.3.3. Collinear vectors:

Vector having a common line of action are called as collinear vectors.

There are of two types of collinear vectors.

(a) Parallel vectors ($\theta=0^0$):-

Two vectors acting along same direction irrespective of their magnitude are called as parallel vectors. Vectors $\vec{A}$ and $\vec{B}$shown in fig. 2.3 are parallel vectors. Angle between them is zero.



(b) Anti-parallel vectors(θ=180^0):-

Two vectors acting along opposite direction irrespective of their magnitude are called as anti-parallel vectors. Vectors $\overrightarrow{A}$ and $\overrightarrow{B}$ shown in fig. 2.4 are anti- parallel vectors. Angle between them is 180^0

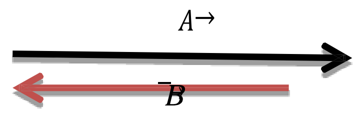


Fig.2.4Anti-Parallelvectors

2.1.3.4. Perpendicular vectors: (θ=90^0):-

Two vectors are said to be perpendicular when they are normal to each other (irrespective of their magnitude). Vectors $\overrightarrow{A}$ and $\overrightarrow{B}$ shown in fig. 2.5 are perpendicular vectors. Angle between them is 90^0

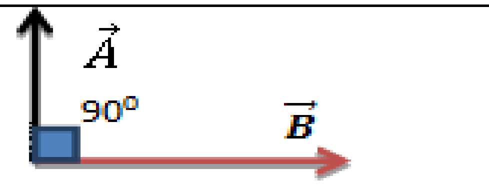


Fig. 2.5 Perpendicular vectors

2.1.3.5. Equal vectors:

Two vectors are said to be equal if they possess same magnitude and direction. All equal vectors are parallel vectors.

2.1.3.6. Negative vectors:

A vector is said to be negative vector of another one if they possess same magnitude and opposite direction.

All negative vectors are anti-parallel vectors.

2.1.3.7. Co-initial vectors:

A number of vectors are said to be co-initial when they have common initial point. Vectors $\overrightarrow{A}$, $\overrightarrow{B}$, $\overrightarrow{C}$, $\overrightarrow{D}$ shown in fig. 2.6 are co-initial vectors, started from a common point P.

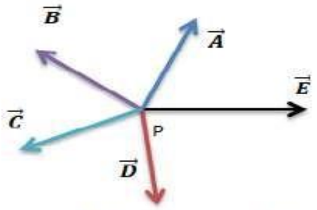


Fig. 2.8 Co-initial vectors

2.1.3.8. Co-planar vectors:

A number of vectors are said to be co-planar when they are lying in the same plane. Vectors $\vec{A}$, $\vec{B}$, $\vec{C}$, $\vec{D}$, $\vec{E}$ shown in fig. 2.9 are co-planar vectors, present in the same plane.

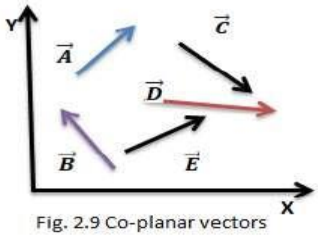


Fig. 2.9 Co-planar vectors

2.1.3.9. Position Vector:

Vectors that indicate the position of a point in a coordinate system is called as position vector. Let point P(x,y,z) present in three coordinate system then $\vec{R}$ is the position vector of point P from the origin O(0,0,0) as shown in the fig 2.10.

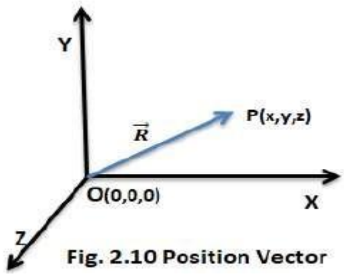


Fig. 2.10 Position Vector

Position vector can be written as $\vec{R} = \hat{\imath}x + \hat{\jmath}y + \hat{k}z$

2.2. Addition and Subtraction of Vectors

Vectors are quantities that have both magnitude and direction. The operations of vector addition and subtraction follow specific rules.

2.2.1.Vector Addition:

Two or more vectors can be added using the head-to-tail method or the parallelogram law.

1. **Head-to-Tail Method:**Place the tail of the second vector at the head of the first vector. The resultant vector is drawn from the tail of the first vector to the head of the last vector.
2. **Mathematical Representation:**If **A** and **B** are two vectors, their sum **R** is given by: **R=A+B**

2.2.2. Vector Subtraction:

Vector subtraction is performed by adding the negative of the vector. If **A** and **B** are two vectors, then:

R=A−B=A+(−B) (where –**B** is a vector equal in magnitude to **B** but in the opposite direction).

2.2.3. Triangle Law of Vector Addition (Statement Only)

According to this law states that “If two vectors are represented in magnitude and direction by the two sides of a triangle taken in order, their resultant is represented in magnitude and direction by the closing side of the triangle taken in the opposite order”.

Let two vectors $\vec{A}$ and $\vec{B}$ acting at a point be represented by two sides $\overrightarrow{OP}$ and $\overrightarrow{PQ}$ of triangle OPQ,taken in same order (Fig. 2.11). According to triangle law of vector addition, the third side $\overrightarrow{OQ}$ represents the resultant vector $\vec{R}$, taken in opposite order.

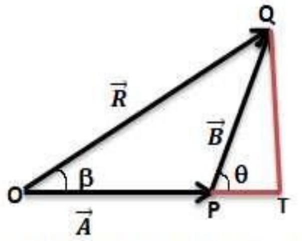


Fig. 2.11 Vector addition by Triangle law

$$\vec{R} = \vec{A} + \vec{B}$$

It can be mathematically proved that:

$$\mathbf{R} = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$\beta = \tan^{-1}\frac{B\sin\theta}{(A + B\cos\theta)}$$

2.2.4. Parallelogram Law of Vector Addition (Statement Only)

According to this law states that "If two vectors acting simultaneously on a body are represented in magnitude and direction by the two adjacent sides of a parallelogram, their resultant is represented in magnitude and direction by the diagonal of the parallelogram drawn from the common point of the two vectors".

Let two vectors $\vec{A}$ and $\vec{B}$ acting at a point be represented by two adjacent sides $\overrightarrow{OP}$ and $\overrightarrow{PT}$ of parallelogram OPTQ, taken in same order (Fig. 2.12). According to parallelogram law of vector addition, the diagonal $\overrightarrow{OT}$ represents the resultant vector $\vec{R}$, passing through that point.

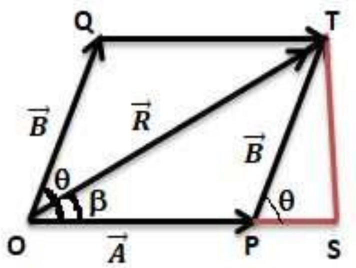


Fig. 2.12 Parallelogram law

$$\vec{R} = \vec{A} + \vec{B}$$

It can be mathematically proved that:

$$\mathbf{R} = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$\beta = \tan^{-1}\frac{B\sin\theta}{(A + B\cos\theta)}$$

2.3. Resolution of Vectors in a plane:

The process of splitting a vector into various parts or components is called "RESOLUTION OF VECTOR". Resolution of a vector is the process of obtaining the component vectors which when combined according to the law of vector addition, produce the given vector

Let $\overrightarrow{OP} = \vec{R}$ be the position vector of point P(x,y) in XY-plane(Fig. 2.13).From P drawthe perpendiculars PQ and PB on X-axis and Y-axis respectively. It makes an angle θwith X-axis.
Let $\hat{\imath}$ and $\hat{\jmath}$are the unit vectors along X-axis and Y-axis respectively.

Consider $\vec{R}$ resolves into two components horizontal component$\vec{R}_x$along X- axis and vertical component$\vec{R}_y$ along Y-axis.
According to triangle law of vector addition in triangle OPQ, we can write

$$\vec{R} = \vec{R}_x + \vec{R}_y$$

$$\vec{R}_x = \hat{\imath} R \cos\theta$$

$$\vec{R}_y = \hat{\jmath} R \sin\theta$$

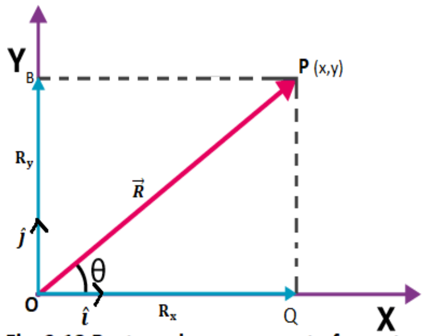


Fig. 2.13 Rectangular component of a vector

2.4. Vector multiplication

There are two ways in which two vectors can be multiplied together.

(i) Scalar Product or Dot product,
(ii) Vector Product or Cross product

2.4.1. Scalar product or Dot product

Dot product between two vectors is defined as the product of their magnitude and the cosine of the smaller angle between them. It is written by putting a dot (•) between two vectors. The result of this product does not possess any direction. Hence it is also called as Scalar product.

Consider two vectors $\vec{A}$ and $\vec{B}$ drawn from a point and inclined to each other at angle θ as shown in the Fig.2.14.

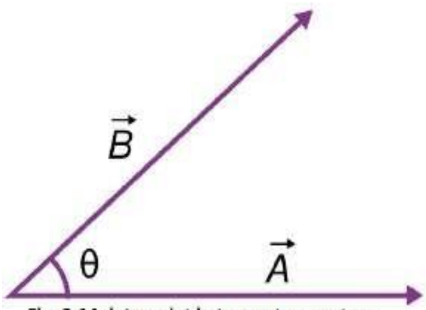


Fig. 2.14 dot produt between two vectors

Dot product of $\vec{A}$ and $\vec{B}$ is given by

$$A\,.\,\vec{B} = AB\cos\theta$$

2.4.2. Cross product or Vector product

Cross product of two vectors $\vec{A}$ and $\vec{B}$ defined as a single vector $\vec{C}$ whose magnitude is equal to the product of their individual magnitude and sine of the smaller angle between them and is directed along the normal to the plane containing $\vec{A}$ and $\vec{B}$.

Consider two vectors $\vec{A}$ and $\vec{B}$drawn from a point and inclined to each other at angle q as shown in the Fig.2.15.

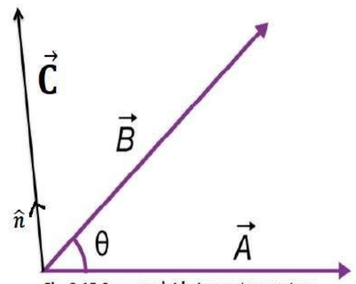


Fig. 2.15 Cross produt between two vectors

Cross product of $\vec{A}$and $\vec{B}$is given by

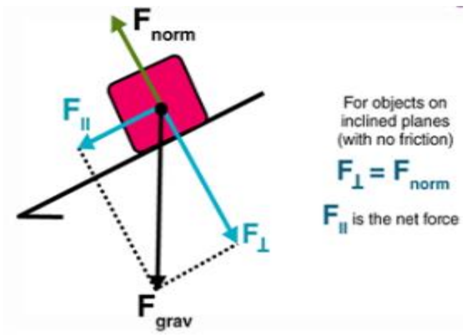
$$\vec{A} \times \vec{B} = \vec{C} = AB \sin\theta \, \hat{n}$$

Where $\hat{n}$is the unit vector of$\vec{C}$directed perpendicular to the plane containing$\vec{A}$ and$\vec{B}$.

2.5. Application of a vector

2.5.1. Inclined Plane:

We can demonstrate a force vector on the inclined plane. Let a block of mass "m" is kept on an inclined plane which makes an angleθ with horizontal direction.



The weight of block "mg" can be resolved into two components. i.e one is inclined plane and another is perpendicular to the inclined plane.

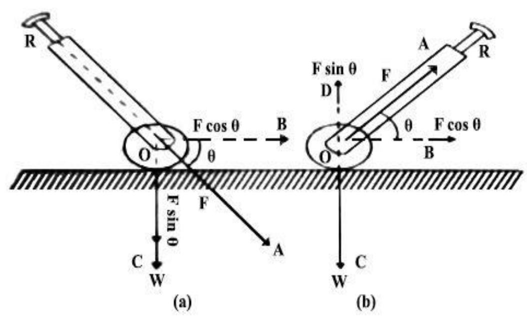
Here in triangle , sinθ =F parallel/ F

F parallel =F sin θ

cosθ = F Perpendicular/F

F perpendicular= F cosθ

2.5.2. Lawn Roller:



When an object is either pulled or pushed on a plane, a frictional force acts between the plane and the object. This retards the motion of the object. The heavier the object, larger is the frictional force. A lawn roller is made moving either by pushing or pulling.

In case of pushing:

Let the weight of the roller be W and the applied force on the handle of the roller be F. Let the force F act at O at angle θ with the horizontal plane [Fig.(a)]. Now, F can be resolved at O into two normal components.

The horizontal component of the force = F Cos θ. which acts along OB in the forward direction and the vertical component of the force – F sin θ, whose direction is along OC. This increases the weight of the roller. So, the total weight of the roller is. (W + R sin θ), which is larger than the actual weight of the roller. Consequently, the frictional force also increases. So, it is difficult to push the roller.

In case of pulling:

Let, the weight of the roller = W and the applied force on the handle = F. The force F acts at O at an angle θ along the horizontal line OB

[Fig. (b)]; This force can be resolved into two normal components. The horizontal component of F is F cos θ. Due to its action, the roller moves in the forward direction.

The vertical component of F is F sin θ, which acts upward along OD. So the total weight of the roller decreases. The weight of the roller is (W — F sin θ), which is less than the actual weight of the roller. The frictional force also decreases, so it becomes easier to pull a roller.

Conclusion: It is easier to pull a lawn roller than to push.

2.6: FORCE:

Force is push or pull which can change or tends to change the state of rest or uniform motion of an object while exerts on it.

According to Newton's law, **Force = mass × acceleration**

In S.I system, its unit is Newton and in C.G.S. system , its unit is dyne.

It is a vector quantity

2.6.1 MOMENTUM:

The product of mass and velocity of an object is known as momentum.

Mathematically, it can be written as

Momentum = mass× velocity

In S.I system, its unit is kg m/s and in C.G.S. system , its unit is gm cm /s .

It is a vector quantity

2.6. 2 CONSERVATION OF LINEAR MOMENTUM:

Conservation of linear momentum is a fundamental principle in physics, stating that the total momentum of an isolated system remains constant if no external forces act on it. In other words, if no external forces are exerted on a system, the system's total momentum before a particular event must be equal to the total momentum after that event. This principle is derived from Newton's laws of motion and applies to a wide range of physical phenomena.

2.6.3. Law of Conservation of Momentum using Newton's Second Law

There are n particles

$m_1, m_2, m_3, \ldots\ldots, m_n$ are the masses of n particles

$v_1, v_2, v_3, \ldots\ldots, v_n$ are the velocities of n particles

Then,

Total linear momentum $= m_1v_1 + m_2v_2 + m_3v_3 + \ldots\ldots\ldots\ldots + m_nv_n$

$$p = p_1 + p_2 + p_3 + p_4 + \ldots\ldots\ldots\ldots\ldots + p_n$$

According to Newton's Second Law of motion:

External Force applied is directly proportional to the change in momentum of the system.

$$F = ma$$

$$F = mdv/dt$$

$$F = d(mv)/dt$$

$$F = dp/dt$$

For isolate system, $F = 0$, then $dp/dt = 0$

Only possible when p is constant because a derivative of a constant is 0.

$$p = \text{constant}$$

$$p_1 + p_2 + p3 + p_4 + \ldots\ldots\ldots\ldots\ldots + p_n = \text{constant}$$

Therefore, If the external force is not applied, then the linear momentum remains constant.

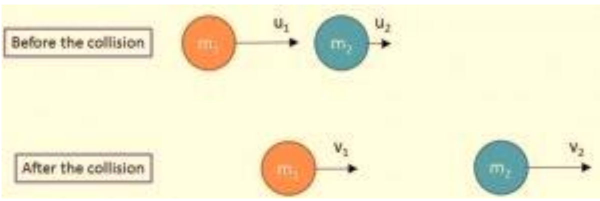
2.6.4. Law of Conservation of Momentum using Newton's Third Law

There are two bodies A and B with mass m_1 and m_2 respectively, initially moving with the velocity u_1 and u_2.

v_1 and v_2 is the velocity of A and B after the collision.

F_{AB} = Force exerted on A due to B

F_{BA} = Force exerted on B due to A



According to Newton's third law of motion,

F_{AB} and F_{BA} are in the opposite direction.

So, $F_{AB} = -F_{BA}$ ....Equation (1)

Impulse (F_{AB}) = $F_{AB} . \Delta t$ = change in momentum of A

$$= m_1v_1 - m_1u_1 \text{ ....Equation (2)}$$

Impulse (F_{BA}) = $F_{BA} . \Delta t$ = change in momentum of B

$$= m_2v_2 - m_2u_2 \text{ .... Equation (3)}$$

From Equation (1),

$$\Rightarrow F_{AB} = - F_{BA}$$

Multiplying both side by Δt,

$$\Rightarrow F_{AB} .\Delta t = - F_{BA} .\Delta t$$

So, $m_1v_1 - m_1u_1 = - (m_2v_2 - m_2u_2)$

or, $m_1v_1 - m_1u_1 = - m_2v_2 + m_2u_2$

$$\therefore m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Total linear momentum (system) before collision = Total linear momentum (system) after collision.

2.6.4. Application:

2.6.4.1.Rocket Launch

When the rocket is launched, the burning fuel ejects from the lower end of the rocket, which forces the rocket machine to move in the opposite direction of the fuel ejected. The mass of the rocket keeps decreasing along with the burning of the fuel due to which the momentum of the rocket keeps increasing. The total momentum of the system, including rocket and fuel, remains same as before repulsion of the rocket.

2.6.4.2.Gun Recoil:

When a bullet is fired, there is a backward force on the gun. According to Newton's third law of motion, every action has an equal and opposite reaction. The total momentum of the recoiled gun and bullet remains zero.

2.7. Impulse:

Impulse is defined as the product of the force applied and the time interval for which the force acts on the body.

It is calculated using the formula, $J = F \times \Delta t$.

2.8. Application:

A cricket player lowers his hand just before catching the ball. This increases the time of impact and decreases the effect of force.

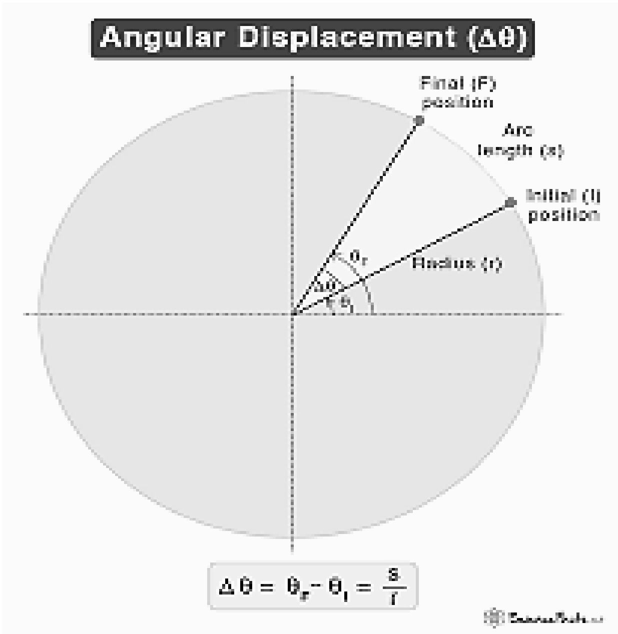
When someone falls from the bed on a cemented floor receives more injuries when compared to falling on a heap of sand. This happens because the sand yields more than the cemented floor, therefore, increasing the impact time and lowering the impact of force.

2.9. Circular Motion:

Motion on a circular path is circular motion. Even a motion on a curved path can be considered a combination of several circular motions. Also, a straight-line motion can be considered as a circular motion of infinite radius.

2.9.1. Angular Displacement (θ):

Angular displacement is the angle turned by a particle moving on a circular path in a certain time. It is a vector quantity and direction of angular displacement is found by using Right Hand thumb rule. Rotate the curl of fingers of right hand in direction of rotation on circular path, then the direction of thumb of right hand gives the direction of angular displacement vector. So, angular displacement **θ** is an axial vector.



We know that angles can be measured in degrees.
They can also be measured in something called Radians
where

$$\textbf{Angle (in radians)} = \frac{\textbf{ArcLength}}{\textbf{Radius}} \qquad \boldsymbol{\theta = \frac{s}{r}}$$

2.9.2. Angular Velocity :

Angular Velocity is the rate of change of angle with respect to time.
Angular Velocity is measured in radians per second, (rad/s).
The symbol for angular velocity is ω (pronounced "omega").

$$\boldsymbol{\omega = \frac{\theta}{t}}$$

If an object is moving in a circle at constant speed, it is accelerating.

This is because while its speed is not changing, its velocity is. Why? Because velocity is defined as speed in a given direction, so if direction is changing, even though speed is not, then the velocity is changing, therefore the object is accelerating.

2.9.3. Relationship between Linear Speed (v) and Angular Velocity (ω)

$$\mathbf{v = r\omega}$$

To derive $v = r\omega$

$$\theta_{(in\ radians)} = \frac{arc\ length}{radius}$$

$$\Rightarrow \theta = \frac{s}{r}$$

$$\frac{\theta}{t} = \frac{s}{tr}$$

{divided both sides by t}

$$\frac{\theta}{t} = \frac{s}{t} \times \frac{1}{r}$$

But $\frac{\theta}{t} = \omega$ and $\frac{s}{t} = v$

$$\omega = v \times \frac{1}{r}$$

$$\Rightarrow v = r\omega$$

2.9.4. Centripetal Force

The force - acting in towards the centre - required to keep an object moving in a circle is called a centripetal force.

2.9.5. Centripetal Acceleration

If a body is moving in a circle the acceleration it has towards the centre is called Centripetal Acceleration.

For both of the definitions above you must refer the object moving in a circle and that the direction is in towards the centre.

Formulae for Centripetal Acceleration and Centripetal Force

$$\boldsymbol{a = \frac{v^2}{r}}$$

but because v = rω we also have

$$a = r\omega^2$$

And because $F = ma$ we get

$$F_c = \frac{mv^2}{r}$$

and also

$$F = mr\omega^2$$

2.9.6, Circular Satellite Orbits

Relationship between Periodic Time and Radius for a Satellite in Orbit*

$$T^2 = \frac{4\pi^2 R^3}{GM}$$

Derivation of formula:

We compare two formulae which we have for Force:

The first is the *Universal Gravitational Force* formula: $F_g = \frac{Gm_1 m_2}{d^2}$

The second is the *Centripetal Force* formula: $F_c = \frac{mv^2}{r}$

Equate both forces (because both equations apply to satellite motion),

Centripetal force = gravitational force

$$\frac{mv^2}{r} = \frac{Gm_1 m_2}{d^2}$$

Cancel one 'm' from both sides

Replace the d^2 in the first formula with r^2 (because in this scenario the distance between the satellite and the planet also corresponds to the radius of the circle that the satellite is tracing out.

Cancel one 'r' both sides
You now have

$$\frac{GM}{R} = v^2$$

Equation (1)

Now v = speed = distance/time.
Distance in this case is the circumference of a circle ($2\pi R$ for circular satellite orbits)

$$\Rightarrow \quad v = \frac{2\pi R}{T} \quad \Rightarrow \quad v^2 = \frac{4\pi^2 R^2}{T^2}$$

Equation (2)

Equating Equations (1) and (2) we get $T^2 = \frac{4\pi^2 R^3}{GM}$

2.9.7. Geostationary Satellites:

These are satellites which remain (are stationery) over one position of the globe, and their orbit is called a Geostationary orbit.

We know that if we want a satellite to remain over a specific spot on the Earth's surface it must have the same periodic time as the Earth (24 hours).

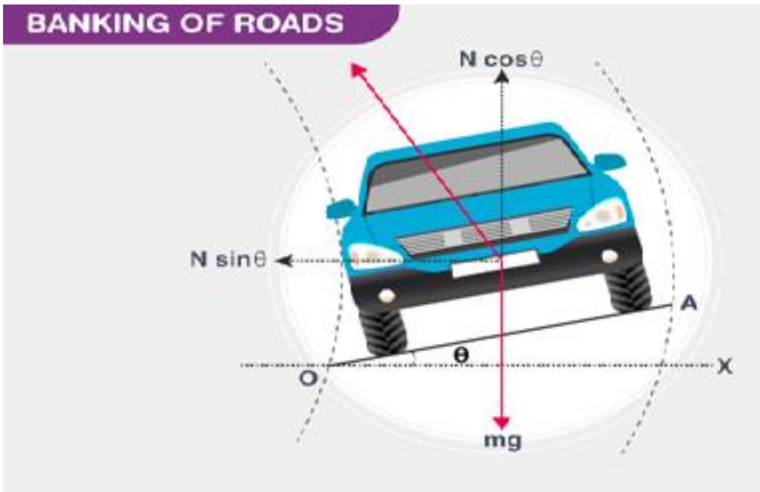
The formula above allows us to calculate the height which the satellite must be at (approx 36,000 km above the equator) in order to have a periodic time of 24 hours).

2.9.8. Banking of Roads:

Banking of roads is defined as the phenomenon in which the outer edges are raised for the curved roads above the inner edge to provide the necessary centripetal force to the vehicles so that they take a safe turn. Now, let us recall,

Centripetal force: It is the force that pulls or pushes an object toward the center of a circle as it travels, causing angular or circular motion. In the next few sections

Let us discuss the angle of banking and the terminologies used in the banking of roads.Another terminology used is banked turn which is defined as the turn or change of direction in which the vehicle inclines towards inside. The angle at which the vehicle is inclined is defined as the bank angle. The inclination happens at the longitudinal and horizontal axis.



2.9.9. Angle of Banking

Consider a vehicle of mass 'm' with moving speed 'v' on the banked road with radius 'r'. Let Θ be the angle of banking, with frictional force f acting between the road and the tyres of the vehicle.

Total upwards force = Total downward force

$N \cos\theta = mg + f \sin\theta$

Where N cosθ = one of the components of normal reaction along the vertical axis

mg = weight of the vehicle acting vertically downward

f sinθ =one of the components of frictional force along the vertical axis

Therefore, $mg = N \cos\theta - f \sin\theta$ ....(eq.^{n}1)

$mv^2/r = N\sin\theta + f \cos\theta$ .... (eq.^{n}2)

Where Nsinθ = one of the components of normal reaction along the horizontal axis

fcosθ = one of the components of frictional force along the horizontal axis

$$\frac{\frac{mv^2}{r}}{mg} = \frac{N\sin\Theta + f\cos\Theta}{N\cos\Theta - f\sin\Theta}$$

(after diving eq.1 and eq.2)

therefore,

$$\frac{v^2}{rg} = \frac{Nsin\Theta + fcos\Theta}{Ncos\Theta - fsin\Theta}$$

Frictional force

$$f = \mu_s N$$
$$\frac{v^2}{rg} = \frac{Nsin\Theta + \mu_s Ncos\Theta}{Ncos\Theta - \mu_s Nsin\Theta}$$
$$\frac{v^2}{rg} = \frac{N(sin\Theta + \mu_s cos\Theta)}{N(cos\Theta - \mu_s sin\Theta)}$$
$$\frac{v^2}{rg} = \frac{(sin\Theta + \mu_s cos\Theta)}{(cos\Theta - \mu_s sin\Theta)}$$
$$\frac{v^2}{rg} = \frac{(tan\Theta + \mu_s)}{(1 - \mu_s tan\Theta)}$$

therefore,

$$v = \sqrt{\frac{rg(tan\Theta + \mu_s)}{(1 - \mu_s tan\Theta)}}$$
$$v_{max} = \sqrt{rgtan\Theta}$$
$$tan\Theta = \frac{v^2}{rg} \text{and} \Theta = tan^{-1}\frac{v^2}{rg}$$

Above is the expression for the angle of banking.

2.9.10.Bending of a Cyclist:

A cyclist provides himself the necessary centripetal force by leaning inward on a horizontal track, while going round a curve. Consider a cyclist of weight mg taking a turn of radius r with velocity v. In order to provide the necessary centripetal force, the cyclist leans through angle θ inwards as shown in figure. The cyclist is under the action of the following forces: The weight mg acting vertically downward at the center of gravity of cycle and the cyclist. The reaction R of the ground on cyclist. It will act along a line-making angle θwith the vertical.

The vertical component R cos θ of the normal reaction R will balance the weight of the cyclist, while the horizontal component R sin θ will provide the necessary centripetal force to the cyclist.

$$R\sin\theta = \frac{mv^2}{r} \text{---------(1)}$$

$$R\cos\theta = mg \text{---------(2)}$$

Dividing eq^n (1) and eq^n (2),

we get

$$\tan\theta = \frac{v^2}{rg} \text{----------(3)}$$

Therefore, the cyclist should bend through an angle $\theta = \tan^{-1}\frac{v^2}{rg}$

It follows that the angle through which cyclist should bend will be greater, if (i) The radius of the curve is small i.e. the curve is sharper (ii) The velocity of the cyclist is large.

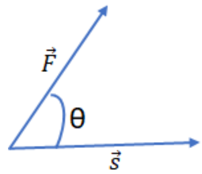
UNIT-3

WORK, POWER AND ENERGY:

3.1.WORK:

Work issaidtobedoneiftheforceappliedonabodydisplacesthebodyandtheforcehasa component along the direction of displacement. Work is a scalar quantity and is the doit product of two vectors Force and Displacement.

Mathematically



W=F.S= FS cosθ

Where, W = work done

F= magnitude of the force

s= magnitude of the displacement

θ=angle between the force and displacement

If θ=0°,then W =$Fscos0°=+Fs$.

Here Force and Displacement are in the same direction and work done is *positive,*

Which means work is said to be done *upon* the body.

Example: An object falling freely under the action of gravity, Kicking a football, A car moving forward etc

If θ=90°,then W=Fscos90°=0,

Here Force and Displacement are perpendicular to each other and no work is done.

Example: **A person carrying a box over his head and walking in the horizontal direction**.

In this case, *work done by the force of gravity* is zero.

If θ=180°,then W=$Fscos180°=-Fs$.

Here Force and Displacement are in the opposite direction and *Negative* work is done means work is done *by* the body.

Example: Work done by the force of friction is negative. Pushing a car up a hill, when it is sliding down, Brakes applied to a moving car, Object pulled over a rough horizontal surface etc.

When the force is applied without any displacement, then also work done is zero.

W = F x 0 = 0

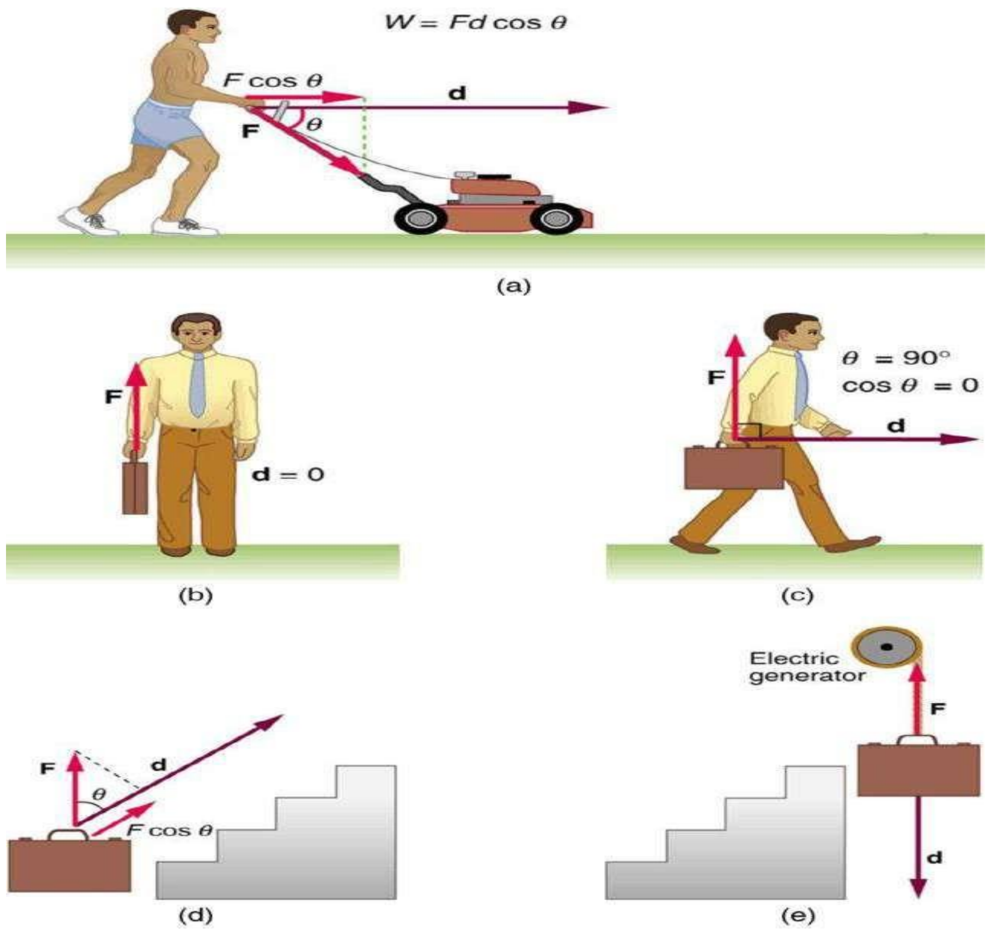
Example: A **person sitting on a chair and studying a book, Pushing a wall etc**

Unit:

The SI unit of work is Joule (J) and C.G.S unit is erg and dimensions of work and energy are same.

Dimension:

$[W]=[F][s][cos\theta]=[MLT^{-2}]\times[L]=[ML^2T^{-2}]$



Example of work (a) positive work- The work done by the force on this lawn mower is F d cosθ, the component of this force is in the direction of the motion ,(b) Zero work- A person holding a brief case does no work on it because there is no motion. (c) Zero work- The person moving the briefcase horizontally at a constant speed does no work on it, as Force and Displacement act Perpendicular to each other.(d) Positive work- Work is done on the briefcase by carrying it up stairs at constant speed, because there is a component of force F in the direction of the motion. (e) Negative Work- Here the work done on the briefcase by the generator is negative, because F and d are in opposite directions.

3.2 FRICTION:

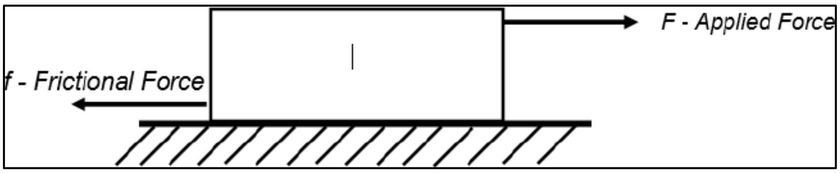
Let us say there is an *almirah* placed on the *floor*. One person tries to push it. He exerts force, the almirah does not move in the beginning. Then, the person increases force little by little and at one point the almirah starts to move.

Let us analysis this situation. When the person is applying force, the force must have some effect (the force must create acceleration). But *apparently* there is no effect. Why is it happening? It is because when the person is applying force, the *floor* is exerting an equal amount of force on the almirah.

Hence, the effect of force is getting cancelled. When the person is increasing the force, the force on the almirah by the floor is increasing too. However, there is a *limit* to the force by the floor. Once, it is reached, the almirah starts to move. However, when the almirah is moving, the floor is still applying force on the almirah. The force tries to oppose the motion of the almirah.

In this example, the force on the almirah by the floor arising because of the contact between them is *frictional force*.In this chapter, we will formally discuss the concept of friction, the types of friction and the laws regarding friction.

Definition: The force which opposes or tends to oppose the relative motion between two surfaces in contact is called as force of friction.



Force of friction is created because of the inter-locking of two surfaces in contact.

3.3TYPES OF FRICTION:

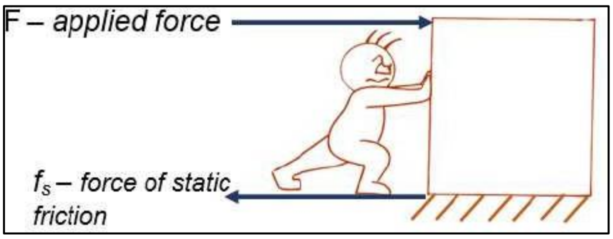
Friction can be classified into four types.

1. Static Friction- Static Friction is the opposing force exists between a surface and object at rest. Example- A book on a table.
2. Kinetic (dynamic) friction-Dynamic Friction is the opposing force created when two solid surfaces slide/ move over one another. Example- writing on paper or pushing a chair across the floor. Walking on the road.
3. Rolling friction-Rolling friction is the opposing force created between moving surfaces when one rolls over another. Example-Car moving on road, Rolling a ball down the lane
4. Fluid friction (viscosity)- Fluid friction is the opposing force created when something tries to move on or through the gas or liquid. Example-Pushing up water backward while Swimming.

   In this chapter we will focus on static and dynamic friction and the laws regarding them.

3.3.1. Static Friction:

- Theforceoffrictionwhichcomesintoplaywhenthereisnorelativemotionbetweentwo surfacesincontactiscalledasforceofstaticfriction.Forceofstaticfrictionisequaland opposite to the applied force till the body is at rest.
- For example, a person or a group of person share trying to push a heavy object. Initially, a small force is applied, and the magnitude of force is increased gradually. The magnitude of static friction increases gradually too. As long as the object is in static condition, the floor exerts an equal and opposite force on the object. As in the below figure, the applied force is towards the right, hence the frictional force is towards be left.



- Static friction is a self-adjusting force.
- The maximum value of static friction is called the limiting friction.

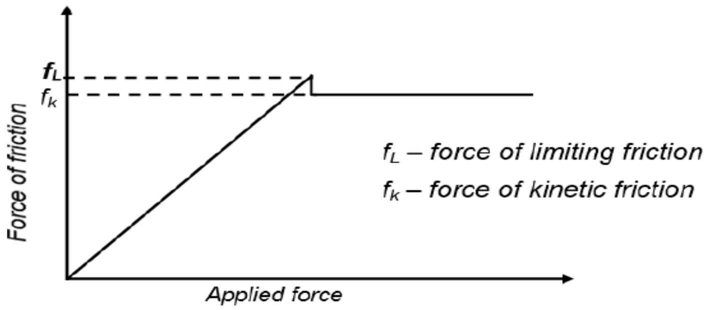
$f_L = \mu_S(1)$

Where f_L = force of limiting friction,

μ_S = co-efficient of static friction,

R= Normal reaction

Once the limiting friction is reached, the body starts to move, and kinetic friction comes to picture.



3.3.2. Kinetic (dynamic) Friction:

The force of friction, which comes into play when there is relative motion between two surfaces in contact is called as force of kinetic friction or dynamic friction or sliding friction. The direction of the frictional force is always opposite to the direction of motion, for which the relative slipping is opposed by the friction.

Hence,$_k = \mu_k R$ -------------------------- (2)

Where f_k =force of kinetic friction,

μ_k=coefficient of kinetic friction,

R=Normal reaction.

3.3.3. LAWS OF LIMITING FRICTION:

Statements about factors upon which the force of limiting friction between two surfaces depends, are termed as laws of limiting friction. They are stated as below.

i. Thedirectionofforceoffrictionisalwaysoppositetothedirectionofmotion.
ii. The force of limiting friction depends on the nature and state of polish of the surfaces in contact and act tangentially to the interface between the two surfaces.
iii. The magnitude of limiting friction f_L is directly proportional to the magnitude of the normal reaction R between the two surfaces in contact.

$$f_L \propto R$$

iv. The magnitude of the limiting friction between two surfaces is independent of the area and shape of the surfaces in contact as long as the normal reaction remains same.

3.3.4. COEFFICIENT OF FRICTION:

- The frictional force (f) is directly proportional to the normal reaction force (R) and the proportionality constant $\mu$is friction. i.e **μ=f/R**

- Hence, the coefficient of friction is defined as the ratio of the friction force to the normal force.
- The coefficient of friction is determined experimentally.
- As the unit and dimension of frictional force and normal force are same, μ is unit and dimensionless.
- The coefficient of friction depends on the nature of the bodies in contact, their material and the surface roughness.

Example1:

A box of mass 30 kg is pulled on a horizontal surface by applying a horizontal force. If the coefficient of dynamic friction between the box and the horizontal surface is 0.25, find the force of friction exerted by the horizontal surface on the box.

Answer:

Given Mass (m) = 30kg ,

μk = 0.25

Normal Reaction(R) = mg

$$f_k = \mu_k R$$
$$\Rightarrow F_k = \mu_k mg$$
$$= 0.25 \times 30 \times 9.8$$
$$= 73.5 Newton$$

Example2:

A body of mass 10kg is placed on a rough horizontal surface at rest. The co-efficient of friction between the body and the surface is μ= 0.1. Find the force of friction acting on the body.

Answer:

Since, the body is at rest, the force of static friction will come into play which is equal to applied force. Since, applied force is zero , the force of static friction is zero.

Example3:

Findtheforceoffrictioninsituationasshowninthebelowfigure.Takeg=10m/s^2

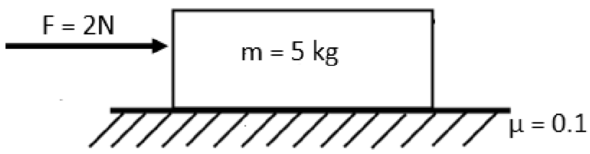


Figure 4.5

Answer:

The magnitude of limiting friction $f_L = \mu R$

$= 0.1 \times 5g$

$= 5N$

We assume that force is smaller than the force of limiting friction

i.e., $F < F_L$

So, the force of static friction = magnitude of applied force = 2N.

3.3.5. METHODS TO REDUCE FRICTION:

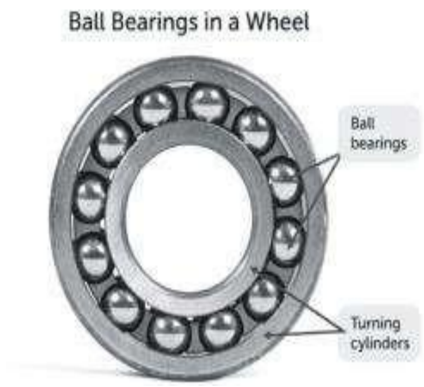
The following methods can be used to reduce friction when friction creates hurdle in the performance of machines or for similar necessary reasons

i. **By polishing or rubbing:**
The roughness of a surface can be reduced by rubbing or polishing it. The polishing makes a surface smooth and reduces friction.

ii. **Lubrication or use of talcum powder:**
Friction can be reduced by using lubricants like oil and grease or talcum powder as they for math in film between different parts of a machine .This film covers up the pores & the lumps present on the surfaces of different parts, and hence improves the smoothness.

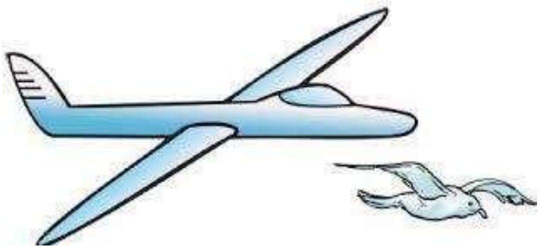
.By converting sliding friction to rolling friction:

Rolling friction is lesser than sliding friction. Hence, ball bearings can be placed between the moving parts of a machine to avoid direct contact between them. This reduces friction.



iii. **Streamlining:**

The objects that move in fluid, for example, bullet train, ship, boat or aeroplane, the shape of the body can be streamlined to reduce the friction between the body and the fluid.



3.4. ENERGY:

Energy is a quantitative physical property that is transferred to some object for carrying network. It is scalar quantities.

In S.I. system , its unit is joule or N-m which is same that as work. In C.G.S. system, its unit is erg or dyne- cm.

3.4.1. TYPES OF ENERGY:

Energy can be divided into two types.

(i) Kinetic energy (K.E) = ½ mv^2

(ii) potential energy (P.E) = m g h

where m = mass of the body,

v = velocity of the body,

g = acceleration due to gravity

h = height of the body

3.4.2: GRAVITATIONAL POTENTIAL ENERGY:

Gravitational potential energy is the energy possessed or acquired by an object due to a change in its position when it is present in a gravitational field. In simple terms, it can be said that gravitational potential energy is an energy that is related to gravitational force or to gravity.

Example:

The most common example that can help you understand the concept of gravitational potential energy is if you take two pencils, one is placed at the table, and the other is held above the table. Now, we can state that the pencil which is high will have greater gravitational potential energy than the pencil that is at the table.

3.4.3. Derivation of Gravitational Potential Energy Equation:

Consider a source mass 'M' is placed at a point along the x-axis; initially, a test mass 'm' is at infinity.

A small amount of work done in bringing it without acceleration through a very small distance (dx) is given by

$$dw = Fdx$$

Here, F is an attractive force, and the displacement is towards the negative x-axis direction, so F and dx are in the same .

Then $$dw = (G\,Mm/x^2)dx$$

Integrating on both sides

$$W = \int_r^\infty \frac{GMm}{x^2} dx$$

$$W = -[\frac{GMm}{x}]_r^\infty = -\frac{GMm}{r}$$

Since the work done is stored as its potential energy U, the gravitational potential energy at a point which is at a distance 'r' from the source mass is given by;

$$U = -G\,Mm/r$$

If a test mass moves from a point inside the gravitational field to the other point inside the same gravitational field of source mass, then the change in potential energy of the test mass is given by;

$$\Delta U = G\,Mm\,(1/r_i - 1/r_f)$$

If $r_i > r_f$ then ΔU is negative.

3.4.4 Expression for Gravitational Potential Energy at Height (h) – Derive ΔU = mgh

If a body is taken from the surface of the earth to a point at a height 'h' above the surface of the earth, then $r_i = R$ and $r_f = R + h$, then,

$$\Delta U = G\,Mm\,[1/R - 1/(R + h)]$$

$$\Delta U = G\,Mm\,h/R(R + h)$$

When h<<R, then $R + h = R$ and $g = GM/R^2$.

On substituting this in the above equation, we get,

Gravitational Potential Energy $\Delta U = m\,g\,h$

⇒ Note:

- The weight of a body at the centre of the earth is zero due to the fact that the value of g at the centre of the earth is zero.
- At a point in the gravitational field where the gravitational potential energy is zero, the gravitational field is zero.

3.4.5 Relation between Gravitational Field Intensity and Gravitational Potential

Integral Form:

$$V = -\int \boldsymbol{E.dr}$$

(If **E** is given and V has to be found using this formula)

Differential Form:

E = -dV/dr (If V is given and E has to be found using this formula)

$$E = -\hat{\imath}\frac{\partial V_x}{\partial x} - \hat{\jmath}\frac{\partial V_y}{\partial y} - \hat{k}\frac{\partial V_z}{\partial z}$$

(Components along x, y and z directions)

Gravitational Potential of a Point Mass

Consider a point mass M, the gravitational potential at a distance ‘r’ from it is given by;

V = – GM/r.

Gravitational Potential of a Spherical Shell

Consider a thin uniform spherical shell of the radius (R) and mass (M) situated in space. Now,

Case 1: If point ‘P’ lies inside the spherical shell (r<R):

As E = 0, V is a constant.

The value of gravitational potential is given by, V = -GM/R.

Case 2: If point ‘P’ lies on the surface of the spherical shell (r=R):

On the surface of the earth, E = -GM/R^2.

Using the relation

$$V = -\int \boldsymbol{E.dr}$$

over a limit of (0 to R), we get,

Gravitational Potential (V) = -GM/R.

Case 3: If point ‘P’ lies outside the spherical shell (r>R):

Outside the spherical shell, E = -GM/r2.

Using the relation

$$V = -\int \boldsymbol{E.dr}$$

over a limit of (0 to r), we get,

V = -GM/r.

3.4.6 Gravitational Potential of a Uniform Solid Sphere

Consider a thin, uniform solid sphere of radius (R) and mass (M) situated in space. Now,

Case 1: If point 'P' lies inside the uniform solid sphere (r < R):

Inside the uniform solid sphere, $E = -G\,Mr/R^3$.

Using the relation of over a limit of (0 to r).

The value of gravitational potential is given by,

$$V = -GM\,[(3R^2 - r^2)/2R^2]$$

Case 2: If point 'P' lies on the surface of the uniform solid sphere (r = R):

On the surface of a uniform solid sphere, $E = -GM/R^2$.

Using the relation of V over a limit of (0 to R) we get,

$$V = -GM/R.$$

Case 3: If point 'P' lies outside the uniform solid sphere (r> R):

Using the relation over a limit of (0 to r), we get,

$$V = -GM/R.$$

Case 4: Gravitational potential at the centre of the solid sphere is given by

$$V = (-3/2) \times (GM/R).$$

3.4.7 Gravitational Self Energy:

The gravitational self-energy of a body is defined as the work done by an external agent in assembling the body from the infinitesimal elements that are initially at an infinite distance apart.

Gravitational self energy of a system of 'n' particles:

Let us consider n particle system in which particles interact with each other at an average distance 'r' due to their mutual gravitational attraction; there are n(n – 1)/2 such interactions, and the potential energy of the system is equal to the sum of the potential energy of all pairs of particles, i.e.,

$$U_s = 1/2G_n(n-1)m^2/r^2$$

Solved Problems

1. Calculate the gravitational potential energy of a body of mass 10 kg and is 25 m above the ground.

Solution:

Given, Mass m = 10 Kg and Height h = 25 m

Gravitational Potential Energy is given as,

$$U = m \times g \times h$$

$$U = 10\ Kg\ 9.8\ m/s^2 \times 25\ m$$

$$U = 2450\ J.$$

Hence, Gravitational Potential Energy of a body is 2450J.

2. If the mass of the earth is 5.98×10^{24} kg and the mass of the sun is 1.99×10^{30} kg, and the earth is 160 million km away from the sun, calculate the GPE of the earth.

Solution:

Given, the mass of the Earth (m) = 5.98×10^{24} Kg

Mass of the Sun (M) = 1.99×10^{30} Kg

The gravitational potential energy is given by:

$$U = -G\ Mm / r$$

$$U = (6.673 \times 10^{-11} \times 5.98 \times 10^{24} \times 1.99 \times 10^{30})/(160 \times 10^{9})$$

$$U = 4963 \times 10^{30}\ J$$

Hence, Gravitational Potential Energy of a body is 4963×10^{30}J.

3. A basketball weighing 2.2 kg falls off a building to the ground 50 m. Calculate the gravitational potential energy of the ball when it arrives below.

Solution:

Given mass (m) = 2.2 kg

Distance (h) =50 m

The gravitational potential energy is given by:

$$U = m\ g\ h$$

$= (2.2 \text{ kg})(9.8 \text{ m/s}^2)(50 \text{ m})$

$= 1078 \text{ J}$

Hence, Gravitational Potential Energy of a body is 1078J.

4: A 2 kg body free falls from rest from a height of 12 m. Determine the work done by the force of gravity and the change in gravitational potential energy. Consider the acceleration due to gravity to be 10 m/s^2.

Solution:

Since, $W = m\, g\, h$

Substituting the values in the above equation, we get

$W = 2 \times 12 \times 10$

$= 240 \text{ J}$

The change in gravitational potential energy is equal to the work done by gravity.

Therefore, Gravitational Potential Energy is 240 J.

3.4.8 MECHANICAL ENERGY:

Mechanical energy is the energy of an object due to its position or motion. It is the basis of physics, as everything around us is driven by mechanical energy. From picking up objects to throwing them, mechanical energy can be seen in action every day. For example, an apple falling from a tree has mechanical energy.

3.4.9 Types of Mechanical Energy:

There are two main types of mechanical energy.

1. Potential Energy: It is the energy stored in an object due to its position. Gravitational potential energy due to Earth's gravity is a common type of potential energy. It depends on the object's height from the Earth's surface.

   For example, an apple in an apple tree has the maximum potential energy. When it falls, its potential energy reduces and becomes zero upon reaching the surface. Apart from gravitational potential energy, other forms are elastic potential energy, electric potential energy, magnetic potential energy, and nuclear potential energy.

2. Kinetic Energy: It is the energy possessed by an object due to its motion. The movement of an object is manifested by its speed. Consider the above example. When the apple is in the tree, it is at rest with zero kinetic energy. When it falls, it gains speed owing to acceleration due to gravity. The kinetic energy increases and reaches a maximum when the apple hits the ground

The types of kinetic energy are electrical energy, thermal energy, radiant energy, and sound energy.

3.4.10 Conservation of Mechanical Energy:

- According to the law of conservation of energy, energy can neither be created nor destroyed. It transforms from one form to another. In the above example, the potential energy of the apple transforms into kinetic energy. Therefore, the sum of potential and kinetic energy remains constant throughout its path. This sum is known as the total mechanical energy. The initial total mechanical energy is the same as the final total mechanical energy.
- Mechanical energy is conserved only when the force acting on an object is conservative. A conservative force does not depend on the path taken to do work. On the other hand, non-conservative and dissipative forces depend on the path taken. In the apple example, gravitational potential energy acts on it, which is a conservative force. If non-conservative forces like friction or air resistance are present, the mechanical energy will get converted into heat energy. In this way, although the mechanical energy is not conserved, the total energy is conserved.

3.4.11Examples of Mechanical Energy:

- A book on a shelf has potential energy due to its height above the surface.
- A moving car possesses kinetic energy due to its motion.
- A baseball on its trajectory has potential energy due to its height and kinetic energy due to its speed.
- A weightlifter lifts a barbell above its head and gives it potential energy.
- A hammer gathers potential energy and transforms it into kinetic energy to hit a nail.
- A loaded dart gun has potential energy as the spring is compressed.
- A roller coaster moves on its track with kinetic energy. When it reaches its highest point, the kinetic energy is converted into potential energy. When it departs, the potential energy is converted back into kinetic energy.
- A swinging pendulum has both potential and kinetic energy. The potential energy is maximum when the pendulum is at two extreme points. The kinetic energy is maximum when it passes through the equilibrium point.
- A wrecking ball is a massive object with potential energy and kinetic energy. When it is swung to a high position, it has potential energy. When it is released, it gains kinetic energy.

3.4.12 CONSERVATION OF MECHANICAL ENERGY FOR FREELY FALLING BODIES:

When a body falls freely, only the gravitational force influences its motion. During the fall the body possesses two types of energies, potential energy and kinetic energy. The sum of these energies is called mechanical energy.Law of conservation of energy states that the energy of a system is always constant.

In other words, we can say that energy can neither be created nor destroyed.
In the case of a freely falling body, it is the mechanical energy of the system that is conserved. Mechanical energy (E) is the sum of the potential energy (U) and the kinetic energy (K) of the freely falling body.

Therefore, E=K+U=constant.

Potential energy is defined as negative of the work done by the force affecting the body. In the case of gravity of earth, if a body of mass m is at height of h then its potential energy is given as

$$U = m\,g\,h.$$

Kinetic energy is the energy possessed by a body when it is in motion.

If a body of mass m is in motion with speed v, then its kinetic energy is equal to

$$K = \tfrac{1}{2} mv^2$$

Let us now prove the conservation of the mechanical energy during a body falling freely.
When a body is falling feely, the only force affecting the motion of the body is the force of gravity exerted by earth, which is equal to F = mg.

Suppose we drop a ball of mass m from height of H.
The potential energy of the ball at height H is

$$U_1 = m\,g\,H \text{.......................(1)}$$

The kinetic energy of the ball at this height is zero because its speed is zero.
Then it will accelerate with the acceleration due to gravity (g).

Suppose that some time t the ball is at the height h.

The potential energy of the ball at this point will be

$$U_2 = m\,g\,h \text{.......................(2)}$$

Hence, the change in potential energy (ΔU) is equal to $U_2 - U_1$.
Let the speed of the ball be v.

Hence, its kinetic energy will be $K_2 = 1/2mv^2$ and

change in kinetic energy $K_2 = 1/2mv^2$

Let us calculate the kinetic energy of the ball at this height.

We know that

$$F = ma$$

$$a = v\,dv/dx$$

Therefore , $F = m.v dv/dx$.

$$\Rightarrow F.\,dx = m.v dv$$

Integrate both the sides.

$$\Rightarrow \int F.dx = \int_{v_1}^{v_2} m.v dv \text{ ....(i).}$$

Here, F=mg and $v_1 = 0$, $v_2 = v$.

Substitute the value of F, v_1, v_2 in equation (i),we get

$\Rightarrow \int m\ g.dx = \int 0vm.vdv$

$\Rightarrow m\ g\ x = 1/2mv^2$ .... (ii).

(where x is the displacement of the ball)

Hence, x=H-h.

Therefore, equation (ii) can be written as,

$mg(H-h) = 1/2mv^2 = \Delta K$ .... (iii)

From equations (1) and (2) we get,

$\Delta U = U_2 - U_1$

$= m\ g\ h - m\ g\ H]$

$= -mg(H-h)$ ....(iv)

Compare (iv) and (v), We get that,

$\Delta K = -\Delta U.$

$E = K + E.$

$\Rightarrow \Delta E = \Delta K + \Delta U.$

But we found that

$\Delta K = -\Delta U.$

Hence, $\Delta E = -\Delta U + \Delta U = 0$

This means that the change in mechanical energy is zero.

Therefore, the mechanical energy is constant or conserved.

3.4.12 Energy Transformation:

Energy transformation or energy conversion is the process of transforming energy from one form to another. According to the law of conservation of energy, energy can neither be created nor destroyed. In other words, energy does not appear out of anywhere and disappears into nothing. It transforms from one form into another.

3.4.13 Types of Energy Transformation:-

As mentioned before, energy can transform from one form into another. Below are the types of energy that one can observe in everyday life.

- Mechanical Energy (including kinetic energy and potential energy)
- Chemical Energy
- Electrical Energy
- Thermal Energy or Heat Energy
- Sound Energy
- Light Energy or Radiant Energy
- Nuclear Energy
- Solar Energy

3.4.14 Energy Transformation Examples

- Here are some examples of energy transformation in daily life.
  An electric fan, blender, and washing machine consist of an electric motor that convert electricalenergy into kinetic energy
- Electric iron, toaster, and stove convert electrical energy into thermal energy
- An electric generator converts mechanical energy into electrical energy
- A television converts electrical energy into sound energy and light energy
- A light bulb converts electrical energy into thermal energy and light energy
- A hairdryer converts electrical energy into thermal energy and sound energy
- The human body digests food and converts chemical energy into mechanical energy enabling muscles to perform work
- A campfire burns wood and converts chemical energy into thermal energy and light energy
- Automobiles use fuel and convert chemical energy into mechanical energy
- The sun transforms nuclear energy into light energy and thermal energy
- Lightning converts electrical energy into light energy, heat energy, and sound energy
- Rubbing hands together converts kinetic energy into thermal energy
- Flashlight converts electrical energy into light energy
- An object speeds up when it falls. Its potential energy is converted into kinetic energy
- A hydroelectric dam converts gravitational potential energy into electrical energy
- A bicycle dynamo converts mechanical energy into electrical energy
- A firecracker transforms chemical potential energy into sound energy and light energy
- A thermoelectric generator is a device that converts thermal energy into electrical energy
- Radio transforms electrical energy into sound energy
- The kinetic energy carried by the wind rotates a windmill to produce electrical energy
- An electrolytic cell converts electrical energy into chemical energy, whereas a voltaic or galvanic cell converts chemical energy into electrical energy.

3.5 POWER:

Power, as a physical quantity, indeed measures the rate of energy transfer. It is defined as the rate the amount at which work is done with respect to time. In other words, power represents of energy consumed or transferred per unit of time.

Mathematically, it is expressed as the ratio of work (W) done to the time taken (t) to do that work.

Power = Work done/ Time

Therefore, it is the rate of energy transfer or work done with respect to time, and the symbol P denotes it.

3.5.1Average Power

If a force performs a certain amount of work within a given time interval, we can compute the average power generated by the force. It is a scalar quantity.

Mathematically, it can be written as $P_{avg}= w/t$

or $P_{avg} = \Delta t\ \Delta E$

3.5.2 Instantaneous power

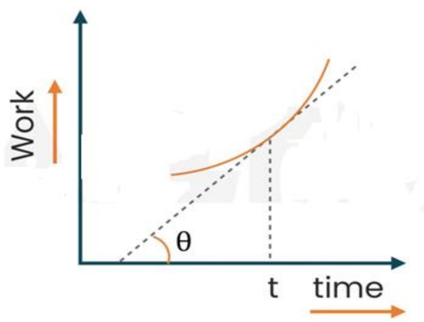
Instantaneous power refers to the power consumed or generated at a specific moment in time. It is given by the scalar or dot product of force and velocity.

Mathematically, it can be written as $P = dt\ dW$

or $P = F.v$

Instantaneous power = slope of work-time curve = $\tan\theta$

When the rate of work done by a body is uniform or constant, the average power and the instantaneous power are inherently equal. This holds true because the average power over any interval of time is calculated by dividing the total work done during that interval by the duration of the interval. Since the rate of work is constant, this average power value remains constant throughout the interval. Consequently, at any moment within that interval, the instantaneous power – which represents the power at a specific instant in time – is also equal to the constant average power.



So, we can state that when the rate of work done by a body is uniform or constant, the average power and the instantaneous power are indeed equal throughout that period.

3.5.3 Unit of Power

The SI unit of power is the joule per second, also known as the watt, named after James Watt.

Another unit of power is Horsepower.

1 Horsepower = 746 watts

(for motors and engines, power is usually measured in Horsepower)

3.5.4 Power Efficiency

Machines are designed to convert energy into useful work; however, because of frictional effects and other dissipative forces, work performed by the machine is always less than the energy supplied to the machine. Thus, we define the efficiency of a machine, which denotes how effective a machine is in converting energy into useful work.

The efficiency of a machine is given by

η=energy input work done

Example 1:

A girl lifts a box of 5 kg up to a height of 20 m for 10 seconds. Calculate the power delivered to the box.

Solution:

Work done by the girl, W = F d = 50 N × 20 m = 1000 J

Power delivered, P = W ⁄ t = 1000 / (10s) = 100 J/s

Hence, the power delivered to the box is 100 J/s.

Example 2:

A motor is used to power a lift that raises a load of bricks weighing 800N to a height of 20m in 40s. What is the minimum power motor needed?

Solution:

Assuming that the bricks are lifted without acceleration, the upward force is equal to the force of gravity 800N. The speed of the bricks is = 40s20m = 0.5 m/s.

Now, power, P = Fv = (800N)(0.5 m/s) = 400 W

If there are no energy losses, e.g. to frictional forces, the motor must have a power output of 400W.

Example 3:

A pump can take out water at the rate of 7200 kg/hr from a 100 m deep well. Calculate the power of the pump, assuming that its efficiency is 50%. (g = 10 m/s2)

Solution:

Given Weight = mg = 7200 kg/hr = 7200×3600 kg/s

Height (h) =100m

Output power = t m g h=7200×3600×10×100=2000 W

Efficiency (η)= input power /output power

Input power = η output power =502000×100=4 kW

3.5.7 POWER AND WORK RELATIONSHIP:

The concepts of power and work form the bedrock of physics, intertwining to describe how energy is transferred and utilized in various phenomena around us. Power, in its essence, quantifies the rate at which work is done or energy is transferred, providing a measure of how swiftly tasks are accomplished. Work, on the other hand, represents the energy transferred to or from an object via the application of force along a displacement. Together, they offer a comprehensive understanding of energy dynamics in physical processes.

The relationship between power and work is succinctly captured by the equation

Power = Work / Time.

This fundamental linkage shows that power is the rate of doing work or, equivalently, the amount of work done over a specific time period. The higher the power output, the more work is done or energy transferred in a shorter amount of time, and vice versa.

These concepts are not merely academic; they have profound implications in our daily lives, from the functioning of simple household appliances to the design of sophisticated engineering systems. Understanding how power and work relate can help us comprehend the efficiency of machines and energy systems, leading to better technology and energy conservation methods.

UNIT-4

ROTATIONAL MOTION:

4.1 TRANSLATIONAL MOTION AND ROTATIONAL MOTION:

When a body does not change its position with respect to time, we say that the body is in rest. But, if a body changes its position with respect to time, we say that it is in motion. There are different types of motion: translational, rotational, periodic, and non-periodic. In this article, we will be discussing the translatory motion and rotational motion.

4.1.1 Translatory Motion:

A type of motion in which all parts of the body move the same distance in a given time is known as the translatory motion. Translatory motion can be of two types: rectilinear and curvilinear.

If a body moves as a whole such that every part of the body moves through the same distance in a given time, then the body is said to be in translatory motion.

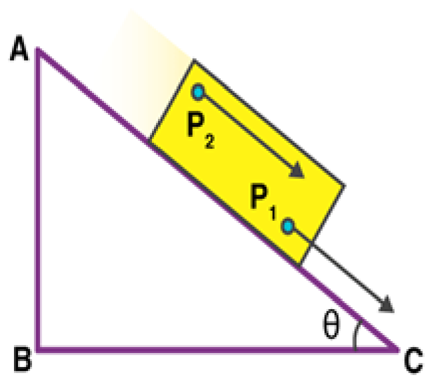
Given below in a table is the difference between rectilinear and curvilinear translatory motion, for your better understanding.

Rectilinear Motion	Curvilinear Motion
When an object in translatory motion moves along a straight line, it is said to be in rectilinear motion	When an object in translatory motion moves along a curved path, it is said to be in curvilinear motion
A car moving along a straight path and the train moving in a straight track are examples of rectilinear motion	A stone thrown up in the air at a certain angle and a car taking a turn are examples of curvilinear motion

4.1.2 Translatory Motion Examples

Let us understand translational motion with the help of examples.

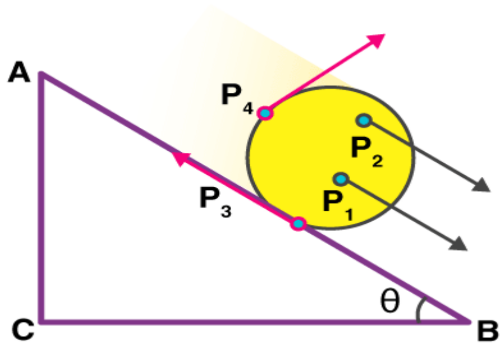
Let's imagine a rectangular block placed on the slanting edge of a right-angled triangle. If the block is assumed to slide down this edge without any side movement, every point in the rectangular block experiences the same displacement, and more importantly, the distance between the points is also maintained. In pure translational motion, every point in the body experiences the same velocity be it at any instant of time. Both the points, P_1 and P_2 undergo the exact same motions.



A rectangular block sliding down the slanting edge of a right-angled triangle covers equal distance in equal intervals of time.
A car moving in a straight line, the path of a bullet out of a gun, etc are examples of translational motion.

4.1.3 Rotational Motion

Now let us imagine a circular block going down the edge of the right-angled triangle. Examining the location and orientation of different points on the cylindrical block will tell us something new. The points on the cylindrical body experience something much different from the rectangular block.



A circular block rolling down the slanting edge of a right-angled triangle experiences different magnitude of velocity in different directions.
As shown by the arrows in the diagram representing the velocity, each point experiences a different magnitude of velocity in a different direction. Here the points are arranged with respect to an axis of rotation. Rotation is what you achieve when you constrain a body and fix it along a straight line. This means that the body can only turn around the line, which is defined as rotational motion. A ceiling fan, a potter's wheel, a vehicle's wheel are all examples of rotational motion.

Say you go to a bowling alley, and throw the bowling ball towards the pins. If you notice closely, you will see that the ball is not just moving forwards i.e. performing translational motion but it is also spinning on itself because of which you can spin and curve the entry of the ball; this motion is categorized as rotational motion. The motion of a rigid body which is not fixed or pivoted is either a pure translational motion or a combination of translational and rotational motion. Rigid bodies are fixed/pivoted experience motion which is rotational.

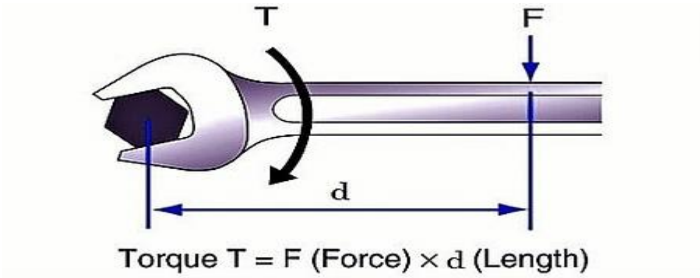
4.1.4 TORQUE:

Torque is the force required to rotate an object around an axis. Just as force causes an object to accelerate in linear kinematics, the torque also causes an object to acquire angular acceleration. It is also referred to as the moment, moment of force, rotational force, or turning effect, depending on the field of study. Torque is a vector quantity.

4.1.5 Causes of Torque:

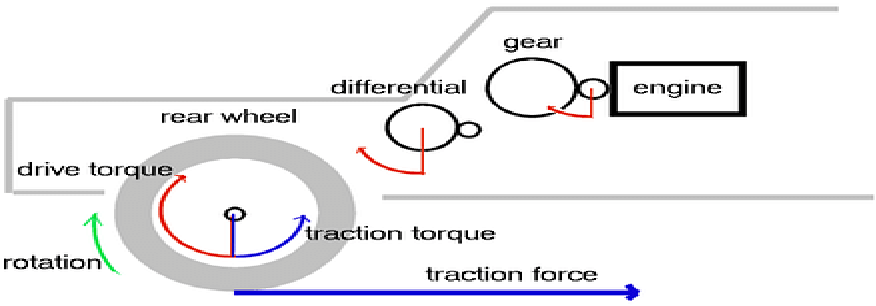
Torque is the force that can cause an object to rotate along an axis.

- Torque is also responsible for **angular acceleration**.
- As a result, torque can be defined as the **rotational equivalent** of a linear force.
- The point at which the item rotates is known as the **axis of rotation**.
- Torque is the tendency of the force to turn or twist the object.
- Torque is described using a variety of terminologies, including **moment** and **moment of force**.
- The **twisting force** that creates motion is referred to as torque.
- The product of the amount of the force acting on the particle and the perpendicular distance of the force applied from the object's axis of rotation is the torque operating on the object.
- **Newton-Meter** (N·M) is the SI unit for torque.



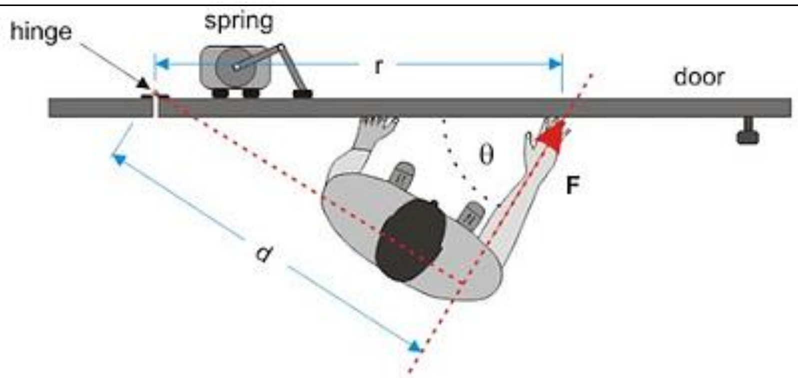
4.1.6 Torque in our Everyday Life:

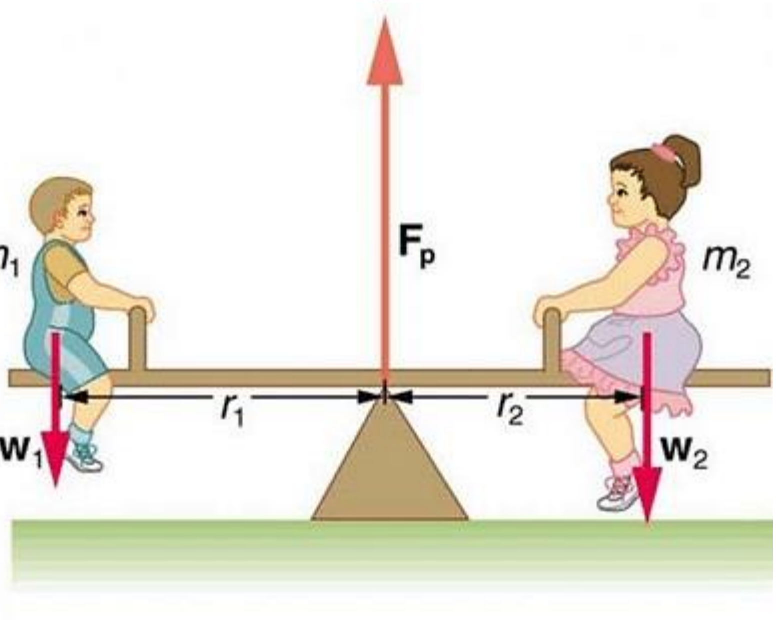
When we hear the word 'torque,' it's almost always in the context of automobiles. Torque is a term that is frequently used to describe a car's power, but what precisely does it mean? Torque is the force exerted by pistons on the crankshaft, which causes it and the wheels to turn in a car.



4.1.7. Torque in automobiles

Torque is a broad physics term that has various applications, despite its association with automobiles. The axis of rotation is the point at which the object is rotating. You use torque on a daily basis without even recognising it. When you just open a locked door, you apply torque three times. The key is turned, the doorknob is turned and the door is pushed to open.





4.1.8 Examples of Torque

In Seesaw many people have observed someone sitting on one end of a seesaw while the other sits on the other end, with one person being heavier than the other. Because the moment arm of a heavier person is shorter than that of the lighter person, the heavier person can reduce torque by sitting closer to the pivot. Because the lever arm is smaller, the torque is reduced, allowing lighter people to lift bigger objects.

4.1.9 Formula of Torque

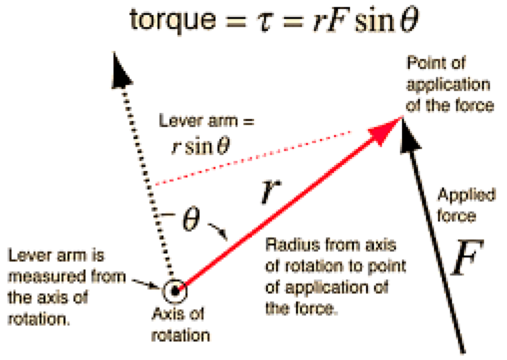
Torque is the multiplication product of the radius vector (from the axis of the rotation to the site of force application) and the force vector. The Torque is represented by symbol τ.

$$\tau = F \times r \times \sin(\theta)$$

Here, F → linear force

r → distance between the axis of rotation and the point at which linear force is applied.

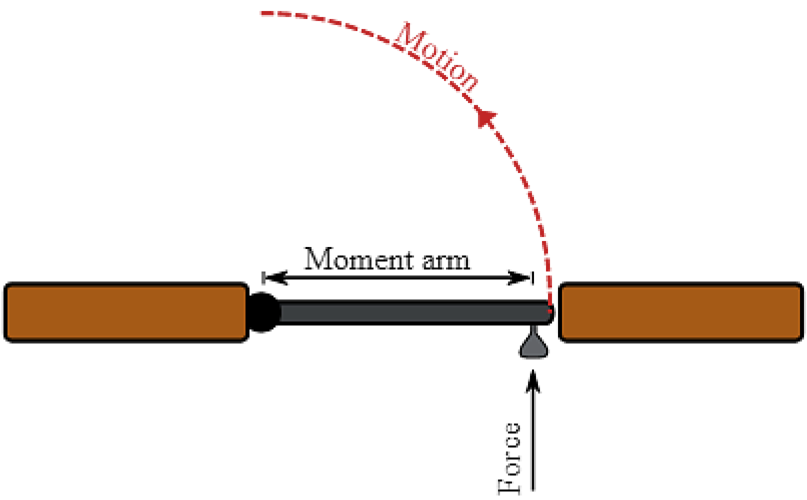
θ → angle between the r and F.



4.1.10 Torque Types

Torque is of two types Static and Dynamic torque.

Static Torque: The torque that does not cause an **angular acceleration** is known as static torque. When a person pushes a closed door, static torque is applied because the door does not rotate despite the force applied.



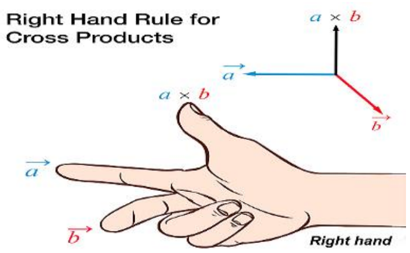
Static Torque Example

Dynamic Torque: The drive shaft of a racing car accelerating from the start line carries the dynamic torque since it must produce an angular acceleration of the wheels if the car is accelerating along the circuit.

4.1.11Torque Determination by Thumb Rule

The right-hand thumb rule is used to determine the direction of the torque vector. The torque vector points in the direction of the thumb if a hand is wrapped around the axis of rotation with the fingers pointing in the force direction.

- Turn your right hand in the direction of the position vector (r and d), then your fingers in the direction of the force, and your thumb in the direction of the torque.
- The direction is reversed when either the direction of r or the direction of F is reversed. If the direction of both r and F is reversed, the direction of torque will remain unchanged. The torque is defined here in terms of a certain point, commonly referred to as the origin. When the same force is applied to a different origin, the torque is different. As a result, determining the source is critical.



- If the force vector is 0° or 180°, the force will not affect the axis rotation. It would either be shoving away from the axis of rotation or shoving towards the axis of rotation because it is in the same direction. In both of these circumstances, the torque value is zero.
- Perpendicular to the position vector, the most influential force vectors for producing torque are θ = 90° or – 90°. This is because it will have the greatest impact on increasing rotation.

4.1.12 Horsepower linked with Torque:

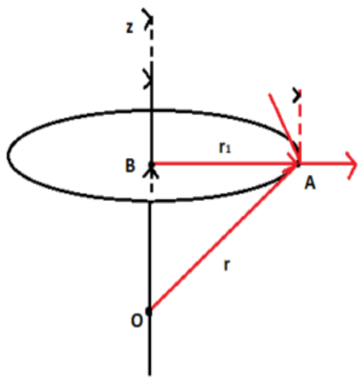
Before purchasing a vehicle, we frequently hear people discussing horsepower and torque, particularly when it comes to racing cars. A horsepower is a **unit of power measurement** that represents the pace at which work is completed.

The term "horsepower" refers to the engine's **total power output**.

To put it another way, if torque is the force that pushes you back in your seat during acceleration, horsepower is the speed reached at the end of that acceleration.

4.2 ANGULAR MOMENTUM:

Angular momentum is basically the product of the moment of inertia of an object and its angular velocity. Furthermore, both the quantities must be about the equal and the same axis i.e. the rotation line.



Angular Momentum = (moment of inertia)(angular velocity)

$$L = I\omega$$

Where L = angular momentum ($kg.m^2/s$)

I = moment of inertia ($kg.m^2$)

ω = angular velocity

4.2.1CONSERVATION OF LINEAR MOMENTUM

We have Newton's second law:

$$\vec{F} = m\vec{a}$$

Now we multiply both the sides by "$\vec{r}$ × ", then we have

$$\vec{r} \times \vec{F} = \vec{r} \times m\vec{a} = d/dt(\vec{r} \times \vec{p})$$

(if we carry out the time derivative, the first term from the product rule is $\vec{v} \times \vec{v}$, which of course is 0.)

This is very similar to the other form of the second law of Newton:

$$\vec{F} = d\vec{p}/dt ,$$

We can read it as: a force provides us the rate of change of the momentum.
A force applied around an axis generates a rate of change of the momentum about the similar axis. This means we identify the left-hand side of the above equation with the torque $\vec{\tau}$ and the quantity inside the brackets of the right-hand side with the angular momentum $\vec{L} = \vec{r} \times m\vec{v}$,
So we can also write the above equation as:

$$\vec{\tau} = d/dt(\vec{L})$$

So in a sense, it's in comparison with the second law of newton that we define this quantity like angular momentum.

4.2.2 APPLICATION OF CONSERVATION OF ANGULAR MOMENTUM:

- Explaining the behaviour of objects in motion, such as the motion of planets and satellites, the spinning of a figure skater, and the motion of subatomic particles.
- Bicycles and motorcycles, flying discs, rifled bullets, and gyroscopes owe their useful properties to conservation of angular momentum.
- Hurricanes form spirals and neutron stars have high rotational rates due to conservation of angular momentum.

4.3. MOMENT OF INERTIA AND ITS PHYSICAL SIGNIFICANT:

4.3.1Moment of Inertia:

Moment of inertia, also known as rotational inertia or angular mass, is a physical quantity that resists a rigid body's rotational motion. It is analogous to mass in translational motion. It determines the torque required to rotate an object by a given angular acceleration. Moment of inertia does not restrict itself to a rigid body only. It also applies to a system of particles rotating about a common axis.

4.3.2 Physical Significance of Moment of Inertia

The physical significance of moment of inertia lies at the heart of understanding rotational motion and the distribution of mass in a given object. Moment of inertia is a fundamental property of matter that quantifies an object's resistance to changes in its rotational motion. When an object rotates, its moment of inertia dictates how difficult it is to either start or stop its rotation or change its rotational speed. It depends not only on the mass of the object but also on how that mass is distributed with respect to the axis of rotation. Objects with larger moment of inertia require more force or torque to be applied to achieve the same angular acceleration as objects with smaller moment of inertia.

The significance of moment of inertia extends beyond just rotational motion. It has practical implications in various fields, including engineering, physics, and even everyday life. Engineers use moment of inertia to design structures and machines that can withstand rotational forces, ensuring stability and safety.

The physical significance of moment of inertia also lies in its ability to quantify an object's resistance to rotational motion. It provides crucial insights into the distribution of mass within an object and influences the rotational behavior, stability, and energy associated with that object. By studying moment of inertia, we gain a deeper understanding of the principles governing rotational dynamics and can apply this knowledge to various real-world applications.

4.4. RADIUS OF GYRATION FOR RIGID BODY:

Radius of gyration or gyradius is an important concept and find its applications in structural engineering, mechanics. It even find its applications in polymer physics where the radius of gyration is used to describe the dimensions of a polymer chain.Radius of gyration also have mathematical definition.

Mathematically, radius of gyration k is the root mean square distance of the particles of the body either from its center of mass or from the axis of rotation, depending on the relevant application.

Radius of gyration generally shows up in two places:

1. **Strength of materials:** Here two dimensional radius of gyration is used and is defined as area property. Here in this case radius of gyration is given by the relation

   $$k=\sqrt{\frac{I}{A}}\;k=IAI$$

   In this case the moment of inertia II is the area moment of inertia. Area moment of inertia is a property of a two-dimensional plane shape which characterizes its deflection under loading. Here in mechanics, we will not concern ourselves with kk calculated using area moment of inertia. This variation finds its applications in engineering (Area Moment of Inertia).

2. **Mechanics:** Here radius of gyration about an axis of rotation is calculated using mass moment of inertia and its formula is given by relation,

   $$k=\sqrt{\frac{I}{M}}\;(1)(1)k=IM.$$

   This equation (1) is the radius of gyration formula for mass moment of inertia. here, MM is mass of the rotating object and II is the moment of inertia about any axis of rotation. In this case it is defined as mass property.

From above explanation we can see that in both the cases radius of gyration means different things i.e., it has different expression. Since we would be studying it in rotational mechanics where we are studying the rotation of objects around a fixed axis of rotation we will only study radius of gyration in context to mass moment of inertia.

Having decided which radius of gyration we have to study here let us now learn about gyradius in detail. When a body or an object is having translational motion the inertia of the body depends only on the mass of the body. In case of rotational motion, moment of inertia depends on two factors

1. Mass of the body
2. Effective distance of its particles from the axis of rotation.

This means that moment of inertia depends on the mass distribution about the axis of rotation.
Radius of gyration of a body is defined about the axis of rotation of the body.

4.4.1 Gyration Radius :

The radius of gyration of a body about its axis of rotation may be defined as the distance from the axis of rotation at which, if the whole mass of the body were concentrated and its moment of inertia about the given axis would be the same as with the distribution of mass.

So, Whatever may be the shape of the body it is always possible to find a *distance from the axis of rotation* at which *whole mass of the body* can be assumed to be *concentrated* and even then its moment of inertia about that axis remains unchanged.
If whole mass of the body is supposed to be concentrated at a distance k from the axis of rotation then

$$I=Mk^2$$
$$=\Sigma mr^2 I$$
$$=Mk^2$$
$$=\Sigma mr^2 I$$
$$k=\sqrt{IM}=\sqrt{\Sigma mr^2 M}k$$
$$=IM$$
$$=\Sigma mr^2 M$$

This quantity k is called radius of gyration of the body about the axis of rotation.
Thus, the radius of gyration of a body, rotating about a given axis of rotation is the radial distance from the axis and when the square of radius of gyration (k) is multiplied by the total mass of the body it gives the moment of inertia of the body about that axis.

4.4.2 Applications of Radius of gyration:-

Radius of gyration can be used to find dynamic quantities of irregular shaped bodies in rotational mechanics. Practically used in airplanes and other automobiles which need a balance but have irregular shape. In such cases radius of gyration is used for the purpose of calculations.

4.5 THEOREMS OF PARALLEL AND PERPENDICULAR AXES

4.5.1 Parallel Axis Theorem

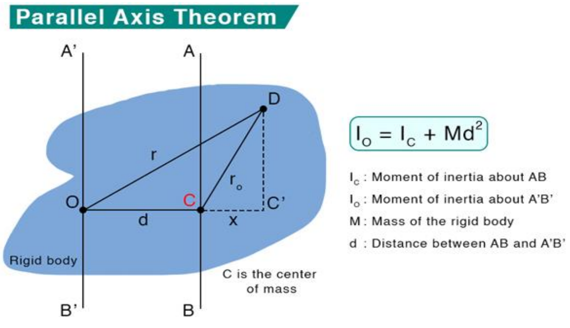
The parallel axis theorem gives a relationship between the moment of inertia of a rigid body about an arbitrary axis and the moment of inertia about an axis passing through the center of mass and parallel to the former. The theorem states, “The moment of inertia of a body about an arbitrary axis is equal to the sum of its moment of inertia about a parallel axis passing through its center of mass and the product of its mass and the square of the distance between the two axes.”

Mathematically, the parallel axis theorem is written as

$$I_O = I_C + Md^2$$

I_C is the moment of inertia about an axis passing through the center of mass.

I_o is the moment of inertia about an arbitrary axis parallel to the axis passing through the center of mass.M is the object’s mass.d is the distance between the two parallel axes.



4.5.2: Derivation

In the image above, AB represents the axis passing through the center of mass C of the rigid body. A'B' represents the axis passing through any arbitrary point D. A'B' is at a distance d from AB. Consider an infinitesimal mass dm at point E.

We have the following quantities.

M: Mass of the body

dm: Infinitesimal mass of point D

I_C: Moment of inertia about AB

I_O: Moment of inertia about A'B'

h: Distance between the two parallel axes AB and A'B'

r_o: Distance between points C and D

r: Distance between points O and D

x: Distance between points C and C'

Consider triangle OC'D.

$$(OD)^2 = (OC')^2 + (C'D)^2$$
$$\Rightarrow r^2 = (d + x)^2 + (C'D)^2$$
$$\Rightarrow r^2 = (d + x)^2 + r_o^2 – x^2$$
$$\Rightarrow r^2 = d^2 + x^2 + 2dx + r_o^2 – x^2$$

$$\Rightarrow r^2 = r_o^2 + d^2 + 2dx$$

Multiplying both sides by dm and integrating

∫r2dm=∫ro2dm+∫d2dm+2d∫xdm∫r2dm
=∫ro2dm+∫d2dm+2d∫xdm

The term ∫xdm∫xdm is zero, since the integral of the moments of infinitesimal masses about the center of mass is always zero when the body is in equilibrium. Therefore, ∫xdm=0∫xdm=0, and we get,

$$I_O = I_C + Md^2$$

Thus, we have derived the parallel axis theorem

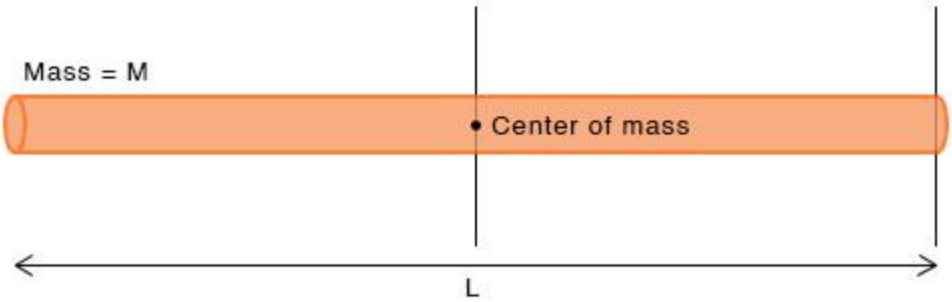
4.5.3 Application of Parallel axis theorem:

The parallel axis theorem can determine the moment of inertia of a given rigid body about any axis. Let us look at a few examples.

1. Uniform Rod

The moment of inertia I_C of a thin rod of mass M and length L about an axis passing through its center is
IC=ML212IC=ML212

Let us apply the parallel axis theorem to find the moment of inertia about its edge I_O. We know that the edge is at a distance L/2 from the center; d = L/2.
IO=ML212+ML24=>IO=ML33IO=ML212+ML24=>IO=ML33

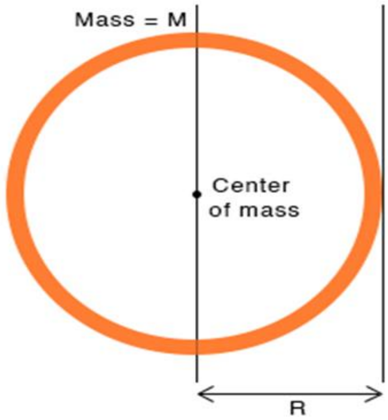


2. Ring

The moment of inertia I_C of a ring of mass M and radius R about an axis passing through its center and perpendicular to the surface is

$$I_C = MR^2$$

The edge is at a distance R from the center. Therefore, the moment of inertia about an axis tangential to the edge and perpendicular to the surface is



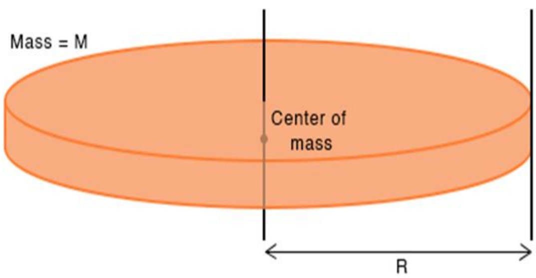
$$I_O = MR^2 + MR^2 = 2MR^2$$

3. Disc

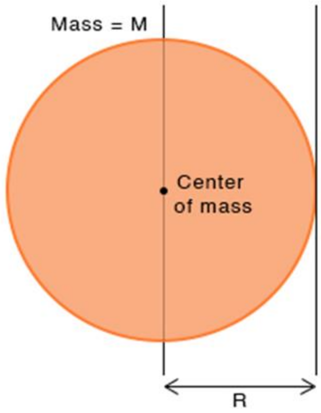
The moment of inertia I_C of a disc of mass M and radius R about an axis passing through its center and perpendicular to the surface is
IC=MR22IC=MR22

The edge is at a distance R from the center. Therefore, the moment of inertia about an axis tangential to the edge and perpendicular to the surface is

IO=MR22+MR2=>IO=32MR2IO=MR22+MR2=>IO=32MR



4. Sphere



The moment of inertia I_C of a solid sphere of mass M and radius R about its central axis is
IC=25MR2IC=25MR2

The surface is at a distance R from the center of the sphere. Therefore, the moment of inertia about an axis tangential to the surface is

I_O=25MR_2+MR_2=>I_O=75MR_2IO=25MR2+MR2=>IO=75MR2

Examples:

1. A body of mass 55 kg has a moment of inertia of 35 kg m^2 along an axis perpendicular to its center of gravity. What is its moment of inertia along a different axis parallel to and 40 cm from the center of mass axis?

Solution: GivenI_C = 35 kg m^2, M = 55 kg, and d = 40 cm = 0.4 m

From the parallel axis theorem,

$I_O = I_C + Md^2$

=> I_O = 35 kg m^2 + 55 kg x (0.4 m)^2

=> I_O = 43.8 kg m^2

2. Find the moment of inertia of a uniform rod with I_C = 0.06 kg m^2, L = 0.3 m, and mass = 1.50 kg about an axis perpendicular to the rod and passing through a point at 1/8 of the length of the rod.

Solution:
Given I_C = 0.06 kg m^2, L = 0.3 m, and mass = 1.50 kg
From the parallel axis theorem

$I_O = I_C + Md^2$
Since the point is at a distance 1/8 of the length of the rod, we have

d = L/2 – L/8 = (3/8) L

=> d = (3/8) x 0.3 m = 0.1125 m

Therefore, the moment of inertia about an axis perpendicular to the rod and passing through the point in question is

I_O = 0.06 kg m^2 + 1.50 kg x (0.1125 m)^2
=> I_O = 0.079 kg m^2

4.5.4 Perpendicular axis theorem:

According to this theorem, "the moment of inertia of a planar body (lamina) about an axis OZ Perpendicular to the pane of the lamina (O being a point in this lamina) is the sum of the moments of inertia about any two mutually perpendicular axes OX and OY, both lying in the same plane", Let I_z = moment of inertia of the lamina about OZ axis.

I_x = moment of inertial about Ox axis.

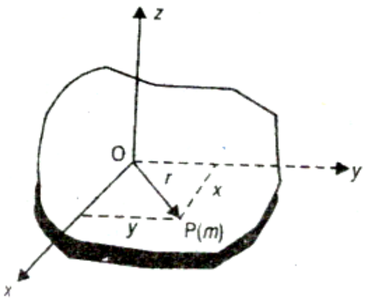
And I_y = moment of inertia about OY axis

Then, $I_z = I_x + I_y$

Proof :

Consider a particle of mass m of the lamina at point p distant r from O. Let (x, y) be point co-ordinates of the point P.(Fig).

The moment



of inertia of the particle about z-axis = mr^2.

Moment of inertia of the whole lamina about z-axis is,

$$I_z = \Sigma mr^2$$

But, $r^2 = x^2 = y^2$

Hence $I_z = \Sigma m\,(x^2 + y^2) = \Sigma mx^2 + my^2$

Now, $\Sigma mx^2 = I_x$ and $\Sigma my^2 = I_y$

Thus, $I_z = I_x + I_y$.

UNIT -5

PROPERTIES OF MATTER:

5.1 ELASTICITY:

When an external force is applied to a rigid body, there is a change in its length, volume (or) shape. When external forces are removed, the body tends to regain its original shape and size. Such a property of a body by virtue of which a body tends to regain its original shape (or) size when external forces are removed is called **elasticity**.

5.2 Unit of Elasticity :

The SI unit for elasticity is the Pascal (Pa). It is defined as force per unit area.
Typically, it is a measure of pressure, which in classical mechanics points to stress.
The Pascal has the dimension $[ML^{-1}T^{-2}]$

5.3 Elastic Stress and Strain:

5.3.1 Stress:

When the body is deformed by the application of external forces, forces within the body are brought into play. Elastic bodies regain their original shape due to internal restoring forces. The internal forces and external forces are opposite in direction. If a force F is applied uniformly over a surface of area A, then the stress is defined as the force per unit area.

Mathematically, it can be written as

Stress = Force/Area

In SI system, the unit of stress is Nm^{-2}

5.3.2 Types of Stress

There are three types of stress:

- Longitudinal stress
- Volume stress or bulk stress
- Tangential stress or shear stress

Longitudinal Stress:

When the stress is normal to the surface area of the body, and there is a change in the length of the body, it is known as longitudinal stress.

Again it is classified into two types:

- Tensile stress
- Compressive stress.

Tensile stress: When longitudinal stress is produced due to an increase in the length of the object, it is known as tensile stress.

Compressive stress: Longitudinal stress produced due to the decrease in length of the object is known as compressive stress.

Volume Stress or Bulk Stress:

If equal normal forces applied to the body causes a change in the volume of the body, the stress is called volume stress.

Tangential Stress or Shear Stress:

When the stress is tangential or parallel to the surface of the body, it is known as tangential or shear stress. Due to this, the shape of the body changes (or) gets twisted.

5.4 Strain:

A body under stress gets deformed. The fractional change in the dimension of a body produced by the external stress acting on it is called strain. The ratio of charge of any dimension to its original dimension is called strain. Since strain is the ratio of two identical physical quantities, it is just a number. It has no unit or dimension.

Strain=Change in dimension initial dimension

Strain is also classified into three types:

- Longitudinal strain
- Volume strain
- Shearing strain or tangential strain

Longitudinal Strain:

The strain under longitudinal stress is called longitudinal strain.

Longitudinal strain=Change in length of the body/ initial length of the body= =$\Delta L/L$

Volume Strain:

The strain caused by volume stress is called volume strain.

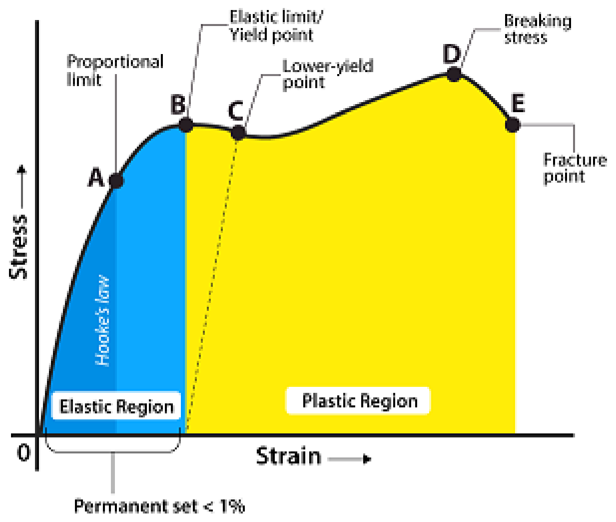
Volume strain=Change in volume of the body /Original volume of the body ==$\Delta V/V$

Shearing Strain:

When a deforming force is applied to a body parallel to its surface, its shape (not size) changes, and this is known as shearing strain. The angle of shear Φ

tanφ=ℓL=displacement of upper face distance between two faces

5.5Stress-Strain Curve:



1. **Proportion limit:** The limit in which Hooke's law is valid, and stress is directly proportional to strain.
2. **Elastic limit:** The maximum stress, which on removing the deforming force, makes the body recover its original state completely.
3. **Lower yield point:** The point beyond the elastic limit at which the length of the wire starts increasing with increasing stress. It is defined as the yield point.
4. **Breaking point or fracture point:** The point when the strain becomes so large that the wire breaks is called the breaking point.

5.6 Elastic Hysteresis

The strain persists even when the stress is removed, and this lagging behind of strain is called elastic hysteresis. This is why the values of strain for the same stress are different while increasing the load and decreasing the load.

5.7 Hooke's Law

According to this law states that "If deformation is small, the stress in a body is proportional to the corresponding strain"; this fact is known as Hooke's law.

Within elastic limit, stress & strain

⇒Stress/Strain=Constant

This constant is known as the modulus of elasticity (or) coefficient of elasticity.

The elastic modulus has the same physical unit as stress. It only depends on the type of material used. It is independent of stress and strain. The modulus of elasticity is of three types:

- Young's modulus of elasticity "γ"
- The bulk modulus of elasticity "B"
- Modulus of rigidity

5.8 Young's Modulus of Elasticity "γ"

Within the elastic limit, the ratio of longitudinal stress and longitudinal strain is called Young's modulus of elasticity (γ).

Mathematically, it can be written as

$$\gamma = \text{Longitudinal stress}/\text{Longitudinal strain}$$
$$= F/A\ell/L$$
$$= FL/A\ell$$

Elastic limit, the force acting upon a unit area of a wire by which the length of the wire becomes double is equivalent to Young's modulus of elasticity of the material of the wire. If L is the length of the wire, r is the radius and is the increase in the length of the wire by suspending a weight (mg) at its one end, then Young's modulus of elasticity of the wire becomes,

Mathematically, it can be written as

$$y = F/A\ell/L$$
$$= FL/A\ell$$
$$= mgL/\pi r2\ell$$

(a) The increment of the length of an object by its own weight:

Let a rope of mass M and length (L) be hanged vertically. As the tension of different points on the rope is different, similarly, the stress, as well as strain, will be different at different points.

- Maximum stress at hanging point
- Minimum stress at a lower point

Consider a dx element of rope at x distance from the lower end, then tension

$$T = (ML) \times g$$

So, $\text{stress} = TA = (ML)xgA$

Let the increase in length of element dx be dy, then

$$\text{Strain} = \text{Change in length}/\text{Original length}$$
$$= \Delta y/\Delta x$$
$$= dy/dx$$

Now, we get stress and strain, then Young's modulus of elasticity "γ"

$$\gamma = \text{Stress}/\text{Strain}$$
$$= (ML)x\, g\, A/dy\, dx$$
$$\Rightarrow (ML)x\, g\, /A\, dx = 1dy$$

The total change in length of the wire is

$$MgLA\int oLxdx=y\int o\Delta ldy$$
$$MgLAL22=y\Delta\ell$$
$$MgL2Ay=\Delta\ell$$

(b) Work done in stretching a wire:

If we need to stretch a wire, we have to do work against its inter-atomic forces, which are stored in the form of elastic potential energy.

For a wire of length (L_0) stretched by a distance (x), the restoring elastic force is

$$F=(Stress)(Area)=y[xL0]A$$

Work required for increasing an element length

$$dW=Fdx=yAL0xdx$$

The total work required to stretch the wire is

$$W=\int 0\Delta\ell Fdx=yAL0\int 0\Delta\ell xdx$$
$$=yAL0[x22]0\Delta\ell$$
$$=yA(\Delta\ell)22L0$$

(c) Analogy of rod as a spring

From the definition of Young's modulus,

$$\gamma=Stress/Strain$$
$$\gamma=FL/A\Delta L$$
$$F=\gamma/A\Delta L/L$$

This expression is an analogy of spring force

$$F=kx$$
$$k=yAL=constant$$

5.9Bulk Modulus (B)

Within the elastic limit, the ratio of the volume stress and the volume strain is called the bulk modulus of elasticity.

$$B=Volume\ stress/Volume\ strain$$
$$=F/A/\Delta V/V$$
$$=\Delta P/\Delta VV$$

5.10 Rigidity Modulus

Within the elastic limit, the ratio of shearing stress (or) tangential stress and shearing strain (or) tangential strain is called the modulus of rigidity.

$$\eta=Shearing\ stress/Shearing\ strain$$
$$=F\ tangential/A\phi$$

Φ =the angle of shear.

5.11Poisson's Ratio

Within the elastic limit, the ratio of lateral strain (or) transverse train and longitudinal strain is called Poisson's ratio. In the case of a circular bar of material, the change in the diameter of the circular bar material to its diameter is due to deformation in the longitudinal direction.

$$Poisson's\ ratio(\sigma)=lateral\ strain/longitudinal\ strain=\beta$$

5.12 PRESSURE AND UNITS:

Pressure is the force applied to the surface of an object per unit area over which that force is distributed. Various units are used to express pressure. Some of these derive from a unit of force divided by a unit of area;

In SI system, the unit of pressure, the Pascal (Pa),

For example, is one Newton per square meter (N/m^2).

Pressure may also be expressed in terms of standard atmospheric pressure.

5.13Pressure Formula:

Pressure acting on a body is the ratio of the perpendicular force to the surface area of the object. The formula that is used to calculate the pressure acting on an area is,

$$P = F / A$$

Where **P** is Pressure
F is Force Applied
A is Surface Area on which force is applied

5.14 Types of Pressure

There are various types of pressure but it can be broadly categorized into four categories.

- Atmospheric Pressure
- Differential Pressure
- Gauge Pressure
- Absolute Pressure

5.14.1Atmospheric Pressure:

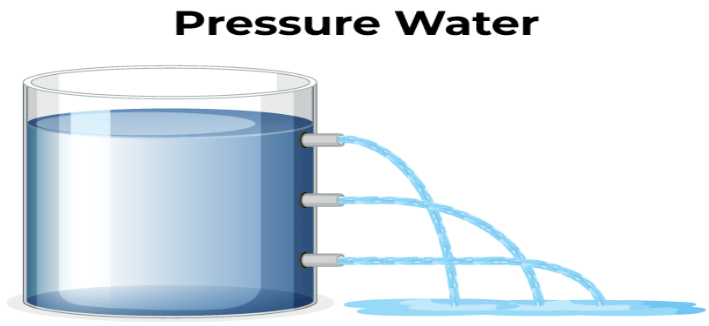
Atmosphere is the envelope of air that surrounds us. Atmospheric air rises hundreds of kilometers above the earth's surface. Pressure exerted by this air is known as atmospheric pressure.

Suppose there is a unit area and a very long cylinder filled with air standing on it, the weight of the air in the cylinder equals the atmospheric pressure. The weight of air in a column with a diameter of 10 cm and a diameter of 10 cm can be as much as 1000 kg. Because the pressure inside our bodies is equal to the atmospheric pressure and cancels out the pressure from outside, we are not crushed under this weight.

Take a high-quality rubber sucker, and It has a structure like a little rubber cup. On a smooth horizontal surface, press it smoothly. When the sucker is pressed, most of the air trapped between its cup and the surface escapes. Because of the pressure of the atmosphere, the sucker sticks to the surface. The applied force must be great enough to overcome atmospheric pressure in order to lift the sucker off the surface. It would not be possible for any human being to pull the sucker off the surface if there were no air at all between the sucker and the surface.

5.14.2 Pressure on Walls of a Container

The container filled with liquid experience pressure which depends upon the height of the water filled in the container. Similarly, the value of pressure experienced by the side walls of the container depends upon the volume of liquid above it. Also, the pressure at any level is the same as the volume of the liquid above that level is always same. Pressure exerted by the liquid on various levels is shown in the image below,



Gases also exert pressure on the wall of the container which contains them. A gas comprises trillions of molecules and every molecule moves in a random direction the moving molecules of gas have some kinetic energy. When these molecules collide with the walls of the container they apply pressure on it.

Example on Pressure Formula

1. **If the force of 10 N acts on an area of 2 m2. Find the pressure acting on that area.**

Answer:

Given,

F = 10 N

A = 2 m2

P = F/A

P = 10/2 = 5

Thus, the pressure acting on the surface is 5 N/m2 .

2.What is the force acting on the body of the pressure acting is 25 N/m2 and its surface area is 5 m2?

Answer:

Given,

P = 25 N/m2

A = 2 m2

P = F/A

25 = F/5

F = 25×5 = 125 N

Thus, the force acting on the surface is 125 N.

3. If the force of 100 N acts on an area of 12 m2. Find the pressure acting on that area.

Solution:

Given,

F = 100 N

A = 12 m^2

P = F/A

P = 100/12 = 8.33

Thus, the pressure acting on the surface is 8.33 N/m^2.

4.What is the force acting on the body of the pressure acting is 220 N/m^2 and its surface area is 11 m^2?
Solution:

Given,

P = 220 N/m2
A = 11 m2
P = F/A
220 = F/11
F = 220×11 = 2420 N

Thus, the force acting on the surface is 2420 N.

5.14.3 Pressure Gauge:

A pressure gauge is a technique of measuring or estimating fluid, gas, water, or intensity of steam in a pressure-powered machine giving assurance regarding leaks or pressure changes that would influence the performance of the system. Any deviation from acceptable standards can truly influence the workings of the system.

Pressure gauges have been utilized for more than 100 years and have been constantly evolving to fit the necessities of new applications. The utilization and execution of pressure gauges have made them a need as increasingly more pressure systems become functional.

5.14.4 Absolute Pressure Gauge:

Absolute pressure gauges are utilized to measure pressure independent of the natural fluctuations in atmospheric pressure. A reference vacuum is attached to the side of the measuring element, which isn't liable to pressure; it has zero pressure with no variety. A diaphragm separates the media chamber from the vacuum chamber and distorts into the vacuum chamber as pressure rises.

5.15: **Fortin Barometer:**

A barometer is an instrument used to measure atmospheric pressure, which plays a crucial role in weather forecasting, altitude determination, and scientific research. Among various types of barometers, the Fortin barometer stands out due to its high accuracy and adjustability. The primary difference between the Fortin barometer and other mercury barometers lies in its unique construction, which allows for more precise readings.

Construction of the Fortin Barometer

The Fortin barometer consists of several key components, each playing a vital role in its functionality:

Glass Tube: A long, sealed glass tube filled with mercury is the core component of the Fortin barometer. The tube is inverted into a mercury reservoir.

Mercury Reservoir: The lower end of the glass tube is submerged in a mercury-filled reservoir. The reservoir is made of glass or metal and includes a flexible leather bag at the bottom.

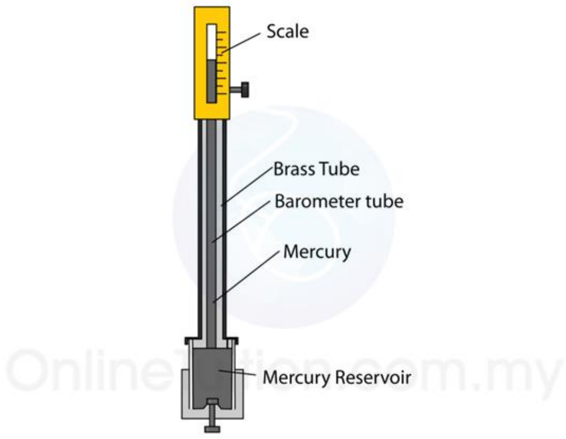
Adjustable Screw (Plunger): The leather bag at the base of the reservoir can be adjusted using a screw mechanism. This screw allows the user to raise or lower the mercury level inside the reservoir, which helps calibrate the barometer and maintain consistent readings.

Pointer or Fiducial Point: A fixed ivory pointer inside the reservoir serves as a reference point for the mercury's surface. The user adjusts the mercury level until it just touches the pointer, ensuring that pressure readings are always taken from the same reference level.

Scale: A graduated scale marked in millimeters or inches is attached alongside the glass tube. This scale provides the pressure reading based on the mercury column's height.

Protective Casing: The entire assembly is usually enclosed in a protective metal or wooden case with transparent windows for easy observation.

Working Principle of the Fortin Barometer



Fortin Barometer

The Fortin barometer operates on the principle of balancing atmospheric pressure with the pressure exerted by a column of mercury. The atmospheric pressure pushes down on the mercury in the reservoir, forcing the mercury to rise inside the sealed glass tube.

The height of the mercury column represents the atmospheric pressure. By adjusting the mercury level to align with the fiducial point, the barometer compensates for temperature changes and ensures consistent readings. The measured height is then read directly from the graduated scale.

Use Mercury in a Fortin Barometer:

Mercury is the preferred liquid for barometers due to several key properties:

High Density: Mercury's high density allows for shorter columns, making the instrument more compact.
Non-Volatile: Unlike water, mercury does not evaporate easily, ensuring long-term stability.
Incompressible: Mercury maintains a consistent volume under varying pressures.
Visible Surface: The metallic surface of mercury provides a clear meniscus for accurate readings.
Calibration and Accuracy:

One of the defining features of the Fortin barometer is its adjustability, which enhances its accuracy. Proper calibration is essential for obtaining precise measurements. The following steps are typically involved in calibrating a Fortin barometer:

Zero Adjustment: The mercury level is adjusted to touch the fiducial point.
Scale Alignment: The scale is set to match the mercury column's height.
Temperature Compensation: Mercury expands and contracts with temperature changes, so temperature corrections may be applied to the final reading.
Regular calibration ensures that the barometer maintains its accuracy over time.

Applications of the Fortin Barometer

Fortin barometers are widely used in various scientific and industrial applications, including:

Meteorology: Accurate atmospheric pressure measurements are essential for weather forecasting and climate studies.
Laboratories: Scientific experiments requiring precise pressure measurements often utilize Fortin barometers.
Altitude Measurement: Barometers can estimate altitude based on pressure differences.
Industrial Calibration: Calibration of other pressure-measuring instruments often relies on the accuracy of Fortin barometers.

Advantages of the Fortin Barometer

- High precision and accuracy
- Adjustable mercury reservoir for consistent readings
- Durable construction
- Long lifespan with proper maintenance
- Reliable for both laboratory and field use

Disadvantages of the Fortin Barometer

- Fragile glass components
- Requires regular maintenance and calibration
- Mercury is toxic and requires careful handling
- Bulky and less portable compared to modern digital barometers

5.16. SURFACE TENSION:

Surface tension is described as the phenomenon that occurs when the surface of a liquid comes into contact with another phase (it can be a liquid as well). Liquids appear to have the smallest possible surface area. The liquid's surface looks like an elastic sheet.

Imagine a line XY (as shown in the figure below) on the independent surface of the fluid in the equilibrium, then at every point on this line, the same force acts in its exact opposite direction. Every point stretched with the same force in both directions.

Thus, in the equilibrium, a force acting on any other per unit length of an imaginary line on an independent surface of the fluid, which is perpendicular to the line and in the direction of the tangent line of the surface, is called Surface Tension.

The **Surface tension definition** reveals that it primarily relies on the attractive forces among the particles of a liquid, as well as the interactions with the adjacent gas, solid, or liquid.

Cohesion and Surface Tension
Surface Tension at liquid molecules is generated by the cohesion force among the atoms of the liquid. The cohesion force is the attractive force between two particles of Solid and Liquid. Cohension force is the force required to hold the solid and liquid particles together.

Surface Tension is the property of the substance because of the cohesion forces. The surface tension resists the change in the structure of the surface.

Surface Tension at Molecular Level

Due to the Cohesion force the water molecule tends to stick together. The water molecule at the bottom layer has various molecules above them to stick but the molecule at the top layer does not have various othermolecules to cling together. Thus, they attach to each other with a larger force and resist any change in their structure.

The molecule inside the body of the liquid experiences the forces from all directions and thus, the net force cancels out each other, whereas the particle at the top layer experiences a strong inward force resulting in the surface tension of the water. Because of this water has one of the highest surface tension among liquids.

Formula for Surface Tension
Mathematically, the surface tension is defined as the force (F) acting on the surface and the length (l) of the surface, so is given as:

$$T = F / l$$

Also, the ratio of the work done (W) and the change in the area of the surface (A) is termed surface tension.

$$T = W / A$$

Unit of Surface Tension
Surface tension is the ratio of the dragging force to the length and thus its SI unit is N/m as force is measured in N and length is measured in m. In the CGS system, its unit is Dyne/cm.

Dimension of Surface Tension
The dimension formula of Force is $[MLT^{-2}]$ whereas the dimension of length is [L] thus, the dimensional formula of Surface Tension is $[M L^0 T^{-2}]$.

Causes Surface Tension:
The effect known as surface tension is caused by the cohesive forces between liquid molecules. Since the molecules at the surface lack like molecules in both directions, they cohere more closely to those specifically aligned with them on the surface. This creates a surface “film” that makes moving an object across the surface more difficult than moving it while fully submerged.

Application of Surface Tension:

Surface tension plays a significant role in everyday life, health, and many industrial processes. There have been a plethora of approaches created to alter surface tension.

- Depending on the surface tension of water, a needle placed in water can float.
- Because of the surface tension of water, mosquito eggs can float.
- Toothpaste contains soap, which helps it spread readily in the mouth by reducing surface tension.
- Antiseptics, such as Dettol, have a low surface tension, allowing them to spread more quickly.

5.17 Viscosity:

Viscosity is the measurement of the resistance of the flowing liquid. Let us learn more about viscosity with an example suppose we take two bowls, one bowl contains water and the other has honey in it, we drop the content of both bowls then we see that water flows much faster than honey which concludes that honey is more viscous than water.

Viscosity is the property of the liquids that prevents liquids from spreading. The force generated due to viscosity is called Viscous Force. Since this force isbetween the layers of liquids, it is alsocalled internal friction. In this article, we will learn about viscosity its formula, measurement, and much more in detail.

Unit of Viscosity:

SI unit for measuring the viscosity of liquid is **Poiseiulle (PI)**. Other units used for measuring viscosity are Newton-Second per Square Metre (Nsm-2) or Pascal-Second (Pas)

Viscosity Dimensional Formula
Dimensional formula of the viscosity is **[ML-1T-1]**

Viscosity Formula
Viscosity is defined as the measure of the ratio of "shearing stress to the velocity gradient of the given fluid". For example when a sphere is dropped into a fluid, then the viscosity of the liquid is determined using the formula:

$$\eta = 2ga2(\Delta\rho) / 9v$$

where,

- ***g** is Acceleration due to Gravity*
- ***a** is Radius of Sphere*
- ***Δρ** is Density difference between fluid and sphere tested*
- ***v** is Velocity of Sphere*

Types of Viscosity
We can measure Viscosity by two methods and the viscosity measured by these two methods is called the types of viscosity. The two types of viscosity are:

- Dynamic Viscosity (Absolute Viscosity)
- Kinematic Viscosity

One way is to measure the fluid's resistance to flow when an external force is applied. This is known as Dynamic Viscosity. And the other way is to measure the resistive flow of a fluid under the weight of gravity. We call this measure of fluid viscosity kinematic viscosity.
The Formula for Dynamic Viscosity is given as

Dynamic Viscosity = Shearing Stress/Shearing Rate Change

The Kinematic Viscosity Formula is given as

Kinematic Viscosity = Absolute Viscosity/Density of the Liquid

Co-efficient of Viscosity:

According to Newton's law of viscosity, the viscous drag, between these layers is,

- Directly proportional to area (A) of the layer **F ∝ A**
- Directly proportional to velocity gradient (dv/dx) between the layers **F ∝ (dv/dx)**

Therefore, it can be written as:

$$F \propto A\ (dv/dx)$$

Lets remove the proportionality sign by introducing a proportionality constant η.

$$F = \eta\ A\ (dv/dx)$$

Here, **η** is called the coefficient of viscosity.
If $A = m^2$ and $dv/dx = 1\ s^{-1}$ then the above expression becomes:

$$F = \eta$$

Thus, the coefficient of viscosity of a liquid is defined as the viscous drag or force acting per unit area of the layer having a unit velocity gradient perpendicular to the direction of the flow of the liquid.

Viscosity Co-efficient Units:
Co-efficient of Viscosity is measured in various units as,

- In the CGS system, the unit of coefficient of viscosity is **dynes s cm-2** or **Poise**
- In the SI system the unit of coefficient of viscosity **N s m-2** or **deca-poise**
- Dimensional formula for the coefficient of viscosity is **[ML-1 T-1]**

Variation of Viscosity:
The coefficient of viscosity depends on the following mentioned factors.

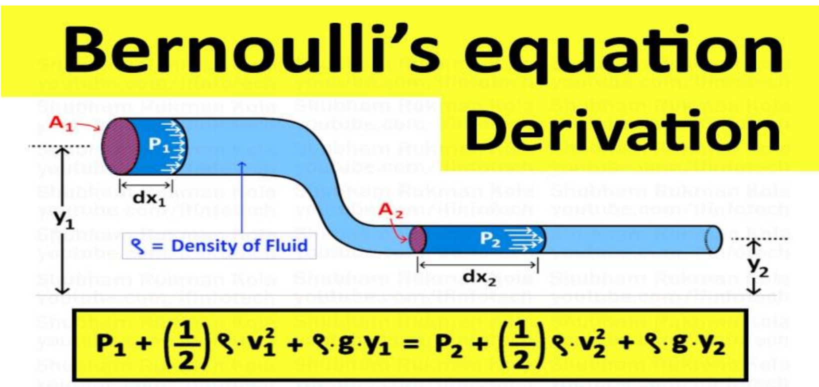
- **Effect of Temperature on Viscosity:** The viscosity of liquids decreases with an increase in temperature. The viscosity of gases increases with an increase in temperatures as $\eta \propto \sqrt{T}$.
- **Effect of Pressure on Viscosity:** The coefficient of viscosity of liquids rises as pressure increases, although there is no relationship to explain the phenomenon thus far.

Applications of Viscosity:
Knowledge of the viscosity of various liquids and gases has been put to use in daily life. Some applications of its knowledge are discussed as under:

- The coefficient of viscosity of organic liquids is used to calculate their molecular weights.
- Knowing the coefficient of viscosity and how it varies with temperature allows us to select the best lubricant for each machine. Thin oils with low viscosity (for example, lubricating oil used in clocks) are utilized in light machinery. Highly viscous oils (for example, grease) are employed in heavy machinery. Viscosity is the most important quality of lubricating oils in lubrication, and it is also highly important in greases, which is frequently overlooked. The resistance to movement is defined as viscosity. Water has a low viscosity because it flows quickly, but honey has a high viscosity.
- The viscosity of a few drugs, such as the numerous solutions used to eradicate moles, has also been reduced to make application simpler. To coat the throat, drug firms provide treatments with a high viscosity yet are still drinkable, such as cough syrup.
- The viscosity of paints, varnishes, and other home items is tightly controlled so that they may be applied smoothly and uniformly with a brush roller.
- Viscosity is an important factor in food preparation and serving. Cooking oils' viscosity may or may not vary as they heat, but many become considerably more viscous when they cool. When fats are cold, they become solid because they are viscous when heated.
- To function properly, manufacturing equipment needs the use of appropriate lubricant. Too viscous lubricants can block and block pipes. Lubricants that are excessively thin provide insufficient protection for moving components.
- Coating viscosity is one of the important characteristics that determine the success of the coating technique. Because the uniformity and repeatability of the coating operation are frequently connected to the viscosity of the coating, it is an important parameter to regulate.

5.18Bernoulli's Theorem

When an incompressible and non-volatile fluid i.e. an ideal fluid flows in a torrent stream in a tube, the total energy of its unit volume or unit mass is fixed at each point of its path. This is called **Bernoulli's theorem.**



The theorem is written in the form of equations and the equations are as follows:
For Unit Volume:

$$P + 1/2dv_2 + dgh = Constant$$

For Unit Mass:

$$P/d + 1/2\ dv_2 + gh = Constant$$

where,

- **P** is the Pressure
- **d** is the Density
- **v** is the Velocity of Flowing Fluid
- **g** is the Gravitational Acceleration
- **h** is the Height of Water from Earth

5.19 Thermal velocity:

Thermal velocity or thermal speed is a typical velocity of the thermal motion of particles that make up a gas, liquid, etc. Thus, indirectly, thermal velocity is a measure of temperature. Technically speaking, it is a measure of the width of the peak in the Maxwell–Boltzmann particle velocity distribution. Note that in the strictest sense thermal velocity is not a velocity, since velocity usually describes a vector rather than simply a scalar speed.

Since the thermal velocity is only a "typical" velocity, a number of different definitions can be and are used.

Taking to be the Boltzmann constant, the absolute temperature, and the mass of a particle, we can write the different thermal velocities:**5.20 Stokes' Law:**

Stokes' law is a mathematical equation for the drag force experienced by small spherical particles passing through a viscous fluid medium. It deals with the resistive (friction) force applied to a body under the action of gravity as it is dropped into a fluid – liquid or air.

According to Stokes' law, the drag force F_d experienced by a spherical particle flowing through a viscous fluid is given by the following formula.

$$F_d = 6\pi\eta rv$$

Where,

η is the viscosity of the fluid

r is the radius of the particle

v is the velocity of the particle relative to the fluid

SI Unit: Newton or N

5.21 Hydrodynamics:

Hydrodynamics is the study of liquids in motion. Specifically, it looks at the ways different forces affect the movement of liquids. A series of equations explain how the conservation laws of mass, energy, and momentum apply to liquids, particularly those that are not compressed.

5.22 Fluid motion:

- Fluid Flow is a part of fluid mechanics and deals with fluid dynamics. It involves the motion of a fluid subjected to unbalanced forces. This motion continues as long as unbalanced forces are applied.

- For example, if you are pouring water from a mug, the velocity of water is very high over the lip, moderately high approaching the lip, and very low at the bottom of the mug. The unbalanced force is gravity, and the flow continues as long as the water is available and the mug is tilted.

Types of Fluids:

Ideal fluid

A fluid is said to be ideal when it cannot be compressed and the viscosity doesn't fall in the category of an ideal fluid. It is an imaginary fluid which doesn't exist in reality.

Real fluid

All the fluids are real as all the fluids possess viscosity.

Newtonian fluid

When the fluid obeys Newton's law of viscosity, it is known as a Newtonian fluid.

Non-Newtonian fluid

When the fluid doesn't obey Newton's law of viscosity, it is known as Non-Newtonian fluid.

Ideal plastic fluid

When the shear stress is proportional to the velocity gradient and shear stress is more than the yield value, it is known as ideal plastic fluid.

Incompressible fluid

When the density of the fluid doesn't change with the application of external force, it is known as an incompressible fluid.

Compressible fluid

When the density of the fluid changes with the application of external force, it is known as compressible fluid.

UNIT – 6

HEAT AND THERMOMETRY:

6.1 Heat:

Heat is defined as **energy that is transferred from one body to another due to a difference in temperature**. When two bodies at different temperatures come into contact, heat flows from the hotter body to the colder one. In essence, heat is the energy associated with the motion of particles within an object, which we perceive as temperature.

6.2 Temperature:

Temperature is a physical quantity that quantitatively expresses the attribute of hotness or coldness. Temperature is measured with a thermometer. It reflects the average kinetic energy of the vibrating and colliding atoms making up a substance.

Thermometers are calibrated in various temperature scales that historically have relied on various reference points and thermometric substances for definition. The most common scales are the Celsius scale with the unit symbol °C (formerly called *centigrade*), the Fahrenheit scale (°F), and the Kelvin scale (K), with the third being used predominantly for scientific purposes. The kelvin is one of the seven base units in the International System of Units (SI).

Absolute zero, i.e., zero kelvin or −273.15 °C, is the lowest point in the thermodynamic temperature scale. Experimentally, it can be approached very closely but not actually reached, as recognized in the third law of thermodynamics. It would be impossible to extract energy as heat from a body at that temperature

Symbol and Units:

The usual symbol for heat is 'Q'. The standard unit of heat in the International System of Units (SI) is the joule (J). Another unit is the calorie (cal), with 1 calorie being the amount of heat required to raise the temperature of 1 gram of water by 1 degree Celsius at atmospheric pressure. The British thermal unit (BTU) is another common unit.

Sign Convention:

In physics equations, the sign convention for heat transfer indicates the direction of energy flow. Heat absorbed by a system is positive ($Q > 0$), indicating that the system's internal energy increases. Conversely, heat released by a system is negative ($Q < 0$), signifying a decrease in internal energy.

Difference Between Heat and Temperature:

Heat and temperature are closely related but distinct concepts. Temperature is a measure of the average kinetic energy of the particles in a substance, and it dictates the direction of heat transfer. Heat, on the other hand, is the transfer of energy due to a temperature difference. It is the process of energy movement, while temperature is a state function that describes a system's thermal state.

6.3 Matter Gains and Loses Heat :

Matter gains or loses heat through the processes of conduction, convection, and radiation:

- **Conduction** is the transfer of heat through a material without the material itself moving. It occurs best in solids.

- **Convection** involves the movement of a fluid (liquid or gas), carrying heat with it. For example, in boiling water, hot water rises and cooler water descends.

- **Radiation** is the transfer of heat through electromagnetic waves and can occur in a vacuum (like heat from the sun reaching Earth).

6.3.1 Heat Formula Using Heat Capacity

One method of calculating the amount of heat transfer to or from a substance is using its heat capacity. Heat capacity is the amount of heat required to change the temperature of a certain quantity of the substance by one degree. The equation is:

$Q=mc\Delta T$

Where:

- Q is the heat added or removed,

- m is the mass of the substance,

- c is the specific heat capacity (energy required to raise the temperature of 1 kg of the substance by 1 degree Celsius),

- ΔT is the change in temperature.

Example : For instance, calculate the heat required to raise the temperature of 2 kg of water from 20°C to 100°C:

Given: $m = 2$ kg (mass of water),

$c = 4.186$ kJ/kg°C (specific heat capacity of water),

$\Delta T = 100°C - 20°C = 80°C$

The heat Q required is: $Q = 2 \text{ kg} \times 4.186 \text{ kJ/kg°C} \times 80°C = 669.76 \text{ kJ}$

6.3.2 Thermal Equilibrium and the Zeroth Law of Thermodynamics:

Thermal equilibrium is a concept where two objects in contact do not exchange heat, as they are at the same temperature. The Zeroth Law of Thermodynamics states that if two systems are in thermal equilibrium with a third system, they are also in thermal equilibrium with each other. This law forms the basis of temperature measurement and underscores the fundamental nature of temperature as a property of matter.

6.3.3 Heat and Entropy

The relationship between heat transfer, entropy, and the Second Law of Thermodynamics illustrates a fundamental principle of the natural world: energy tends to disperse and systems move towards a state of higher entropy.

Entropy, denoted as S, is a measure of disorder or randomness in a system. In thermodynamic terms, it quantifies the number of ways a system can be arranged, often interpreted as a measure of uncertainty or the dispersal of energy within a system.

The mathematical expression for entropy change, ΔS, when a quantity of heat Q is added or removed from a system at a temperature T, is:

$\Delta S = TQ$

$Q = \Delta S / T$

This formula applies when the heat transfer occurs at a constant temperature.

6.3.4. Second Law of Thermodynamics

The Second Law of Thermodynamics states that the total entropy of an isolated system can never decrease over time, and is constant if and only if all processes are reversible. In other words, the universe tends toward greater disorder or entropy.

6.3.5 Heat Transfer and Entropy

When heat transfers from one body to another, the entropy of the system changes. Heat flows naturally from a higher temperature body to a lower temperature body. In this process:

- The entropy of the body losing heat decreases.
- The entropy of the body gaining heat increases.
- The net change in the universe's entropy is positive.

6.3.6 Heat and Enthalpy

Enthalpy, denoted as H, is a measure of the total heat content of a thermodynamic system. It's useful for calculating the heat change in processes occurring at constant pressure. Enthalpy is defined as:

$H = U + PV$

Where:

- U is the internal energy of the system,
- P is the pressure,
- V is the volume.

For processes at constant pressure, the change in enthalpy ΔH equals the heat absorbed or released:

$$\Delta H = Q_p$$

Where Qp is the heat transfer at constant pressure.

6.3.7 Specific heat:

The definition of specific heat capacity of any substance is “the quantity of heat required to change the temperature of a unit mass of the substance by 1 degree”. This is articulated as:

Specific Heat Capacity =Energy Required Mass×∆T

As it indicates the resistance of a material to an alteration in its temperature, specific heat capacity is a type of thermal inertia. Specific Heat Capacity Formula is also communicated in relation to the quantity of heat Q.

C=Qm×∆T

Specific heat capacity in terms of heat capacity is conveyed as

Specific Heat Capacity =Energy Required Mass×∆T

6.3.8 Scales of temperature:

Scale of temperature is a methodology of calibrating the physical quantity temperature in metrology. Empirical scales measure temperature in relation to convenient and stable parameters or reference points, such as the freezing and boiling point of water.

6.3.9 Convert Between Temperatures:

- Celsius to Fahrenheit: Multiply the temperature by 2 and then add 30
- Fahrenheit to Celsius: subtract 30 from the temp. and then divide by 2
- Kelvin to Fahrenheit: Subtract 273.15, multiply by 1.8, then add 32
- Fahrenheit to Kelvin: Subtract 32, multiply by 5, divide by 9, then add 273.15
- Kelvin to Celsius: Add 273
- Celsius to Kelvin: Subtract 273

6.4.TYPES OF THERMOMETER AND THEIR USES:

The need for precise temperature measurement has led to the development of various types of thermometer. These thermometers differ in design, functionality, and areas of application.

Mercury Thermometers

Once a staple in temperature measurement, mercury thermometers are known for their accuracy. However, due to the toxicity of mercury, their use has declined in favor of digital alternatives.

Digital Thermometers

Digital thermometers have replaced traditional mercury thermometers in many applications. They are commonly used in medical settings and can provide quick and accurate readings.

Infrared Thermometers

Infrared thermometers allow for non-contact temperature measurement, making them ideal for industrial and medical use.

Thermometer Classifications

There are primarily three types of thermometer widely used across industries: contact, non-contact, and hybrid thermometers.

Contact Thermometers
These thermometers need physical contact to measure temperature. Examples include thermocouples, resistance temperature detectors (RTDs), and thermistors.

Non-Contact Thermometers
Non-contact thermometers use infrared radiation to measure temperature from a distance, making them ideal for hazardous environments. **Hybrid Thermometers**
These thermometers combine features of contact and non-contact devices, enhancing their versatility.

6.5 Measuring Instruments and Their Uses
In addition to thermometers, there are 10 measuring instruments and their uses that are critical in scientific and industrial settings:

- Thermocouple – Industrial temperature measurement
- Resistance Temperature Detector (RTD) – High-precision temperature monitoring
- Infrared Thermometer – Non-contact temperature readings
- Hygrometer – Measures humidity
- Barometer – Measures atmospheric pressure
- Manometer – Measures gas pressure
- Pyrometer – Measures high temperatures in furnaces
- Thermistor – Detects temperature changes in electronics
- Data Logger – Records temperature trends

Glass Thermometer – Traditional temperature measurement